



It is not the hand that writes, but the thought: Gestures of prospective mathematics teachers

No es la mano la que escribe, sino el pensamiento: Gestos de los futuros profesores de matemáticas

Não são as mãos que escrevem, mas o pensamento: Gestos e perspectivas dos professores de matemática

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Abstract

In this text, four examples of gestures produced and arising from an investigation with mathematics undergraduate students are presented. Gesticulations were used to convey the concepts of point, line, and plane in conversations about the spatial geometry of position. **[Objective]** Thus, we aim to share an analysis of the gestures produced by prospective mathematics teachers to indicate the primitive entities of geometry. **[Methodology]** The data come from a research study involving students aged between 19 and 23 years as participants at a public university in the state of São Paulo, Brazil. This research is qualitative. The analysis of the gestures, as well as their possible meanings, was conducted through an attentive and detailed examination of the video recordings of the two meetings held for each of the six pairs of undergraduates, facilitated by teaching experiments. **[Results]** As a result, we noticed that the same geometric notion was communicated through identical gestures at different times. It was also found, in line with the assumed theoretical framework, that gestures are an integral part of the discourse. Finally, we verified the presence of three gestures produced sequentially, one after the other, to indicate notions of point, line, and plane. **[Conclusions]** These considerations are relevant and show the power of gestures in the sense that they can provide clues to how students are expressing and/or learning concepts of greater complexity.

Keywords: Mathematics education; meanings of gestures; primitive entities of geometry; spatial geometry.

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Resumen

En este artículo se presentan cuatro ejemplos de gestos producidos y emergentes de una investigación con estudiantes de licenciatura en matemáticas. Las gesticulaciones se emplearon para comunicar las ideas de punto, línea y plano, en situaciones de diálogo, sobre contenidos de Geometría Espacial de Posición. **[Objetivo]** El propósito es compartir un análisis de los gestos producidos por futuros profesores de matemáticas para señalar las entidades primitivas de la geometría. **[Metodología]** Los datos provienen de una investigación de carácter cualitativo, cuyos participantes fueron estudiantes de entre 19 y 23 años de edad, pertenecientes a una universidad pública ubicada en el estado de São Paulo, Brasil. El análisis de los gestos, así como de sus posibles significados, se realizó a partir de una observación minuciosa de las grabaciones, en video, de dos encuentros desarrollados con cada una de las seis parejas de estudiantes, en el marco de experimentos de enseñanza. **[Resultados]** Se identificó que una misma noción geométrica fue comunicada mediante gestos idénticos en distintos momentos. Asimismo, se constató, en consonancia con el marco teórico adoptado, que los gestos constituyen una parte integral del discurso. Finalmente, se registró la presencia de tres gestos producidos, de forma secuencial, uno tras otro, para representar las nociones de punto, línea y plano. **[Conclusiones]** Estas consideraciones son relevantes y evidencian el potencial de los gestos para ofrecer indicios sobre cómo los estudiantes expresan y aprenden conceptos de mayor complejidad.

Palabras clave: Educación matemática; entidades primitivas de la geometría; geometría espacial; significados de los gestos.

Resumo

Este artigo apresenta quatro exemplos de gestos produzidos e resultantes de pesquisas com alunos de graduação em matemática. Os gestos foram usados para comunicar as ideias de ponto, linha e plano em situações de diálogo sobre o conteúdo de Geometria de Posição Espacial. **[Objetivo]** O objetivo é compartilhar uma análise dos gestos produzidos por futuros professores de matemática para apontar as entidades primitivas da geometria. **[Metodologia]** Os dados são provenientes de uma pesquisa qualitativa, cujos participantes foram alunos com idade entre 19 e 23 anos, pertencentes a uma universidade pública localizada no estado de São Paulo, Brasil. A análise dos gestos, bem como de seus possíveis significados, baseou-se em uma observação minuciosa das gravações em vídeo de dois encontros com cada uma das seis duplas de alunos, no âmbito de experimentos de ensino. **[Resultados]** Verificou-se que a mesma noção geométrica foi comunicada por gestos idênticos em momentos diferentes. Também foi constatado, de acordo com o referencial teórico adotado, que os gestos são parte integrante do discurso. Por fim, foi registrada a presença de três gestos produzidos sequencialmente, um após o outro, para representar as noções de ponto, linha e plano. **[Conclusões]** Essas considerações são relevantes e mostram o potencial dos gestos para fornecer indícios sobre como os alunos expressam e/ou aprendem conceitos mais complexos.

Palavras-chave: Entidades primitivas da geometria; significados dos gestos; educação matemática; geometria espacial.



Introduction

This text is part of a larger study we developed with prospective mathematics teachers at a public state university in the state of São Paulo, Brazil. The main theme of the research was the spatial geometry of position (SGP). Following an attentive examination of the data generated through various methodological resources during the two phases of the investigation, two categories emerged, one of which was titled “Gestures as protagonists of the construction and communication of spatial geometric concepts.” This text brings an excerpt from this category.

We chose to present the dialogues conducted with six pairs of students who participated in the second stage of data collection. They had SGP content as their generating themes and were recorded through video recordings. The transcriptions of conversations between students and the researcher (the first author of this text), as well as the screenshots containing instances where the gestures were produced, comprise the analytical corpus. For this text, we deal with two integrating axes, one, by nature, is spatial geometry, and the other, resulting from data, is gestures. We will address these aspects briefly later.

About geometry, we still notice a very recurrent practice in mathematics classes, in which “[...] The properties of the points of a line are first presented, and we move on to angles, triangles, quadrilaterals, polygons, and circles, these subjects being presented in an axiomatic and abstract way” (Fainguelelnt, 1999, p. 13). In our study, we attempted to avoid this model by asking questions that involved SGP concepts, aiming to promote a spontaneous debate in which participants could express themselves

in various ways, including using the manipulable materials available in the classroom during the teaching experiments (Steffe & Thompson, 2000).

Regarding research involving the teaching and learning of geometry, metric spatial geometry (MEG) is prioritized over SGP (Bispo & Assis, 2021). Perceiving this gap in studies involving the SGP, we agree with Ferner et. al (2016), when he ponders that

[...] It is important to verify the research focused on the teaching and learning of geometry, because the teaching of this field generally focuses on the memorization of formulas, which does not contribute to the development of the ability to abstract, to estimate and compare results, to recognize properties of geometric shapes, essential in solving problems in mathematics itself and in other areas of knowledge (p. 2).

Regarding learning, we agree with D’Ambrósio (2002, p. 51) when he argues that “[...] Learning is not the mere mastery of techniques, skills or the memorization of some explanations and theory”. A teaching of geometry centered on memorizing formulas and theorems contradicts what the author reported earlier. In view of these problems that involve the teaching and learning processes of geometry, it becomes evident that body language can reveal the mathematics that we have in our bodies, without resorting to a traditionalism focused on the use of formulas and memorization of an extensive geometric vocabulary that refers to complex notions.

Regarding gestures: Are they arm and hand movements? Are they intentional? Do they communicate? If they convey a message, what does it mean? What can



it reveal in the context of the discussion of spatial geometric knowledge? These are some questions that we will address in this text. We will not provide a definitive answer to each of them. The reader will be able to deepen and draw their own conclusions.

Regarding the definition of gestures, [Silva and Farias \(2022\)](#) warn that other parts of the body are also gesture-producing. This is the path we have taken for this text. Facial expressions, nodding, the exchange of glances, movements of the arms and hands, as well as the upper part of the body, are forms of communication and/or externalization of mathematical concepts considered in our investigation; therefore, they are considered gestures. According to [Kendon \(2004\)](#), these movements transcend the idea that they are merely bodily actions; they are intentional and convey messages during a conversation. In another study, [Kendon \(1980\)](#) defines gestures as perceptible movements of the hands, face, and body, which occur intentionally to communicate something. Another characteristic noted is that they are produced concurrently with oral verbal language.

According to [Silva and Farias \(2022\)](#), multimodality has been defending the thesis that gestures are forms of expression with a robust connection to language and speech. This, in turn, joins the gesture in such a way that both form links in a chain, united and intertwined. Taking these considerations into account, we were interested in gesture-speech integration in contexts involving SGP concepts.

According to [Suárez \(2011\)](#), the meaning of a gesture accompanied by speech is decoded at once, meaning that the listener, for example, can understand a gesture that indicates the idea of a spatial

geometric concept from the visual symbol that the gesticulation emits.

Gestures, imperceptible to many people, especially to those who perform them, are protagonists in this text. Thus, we aim in this work to share an analysis of the gestures produced by prospective mathematics teachers to indicate the primitive entities of geometry. We sought to reveal the meanings of gestures when participants had to resort to the *point*, *line*, and *plane* elements to expose some property of geometry. These gesticulations were interpreted in the mathematical context. In other situations, they can even convey any other meanings, because, as [Barsalou \(2009\)](#) rightly said, gestures are situated, which means that they can have different meanings, depending on the context and/or theme we are discussing.

Theoretical framework

In this section, we present the perspective of some researchers on the definition of gestures ([Farsani & Villa-Ochoa, 2022](#)). Then, we initiated a discussion to emphasize the synchrony between gesture and speech, for which we posed the following question: Which comes first, gesture or speech? Later, we reserve a part to show the main characteristics of gestural dimensions. Finally, we focus our attention on the meanings of gestures in the context of mathematics education.

Gestures: First impressions

What do we understand by gestures? Could we explain it to another person? Why do we make gestures when talking to someone else and/or talking on the phone? What conception of gestures will we adopt in this text? What paths do we take to arrive at a possible definition that we consider



to be pertinent? We believe it is valid and opportune to present our position on this topic. However, before that, we would like to highlight what some experts have to say. We begin with [David McNeill \(1992, p. 11\)](#), who states that gestures are “spontaneous movements of the arms and hands [...] closely synchronized with the flow of speech”. Are arms and hands the only producers of gestures? We sought out other researchers to explore different perspectives. In [Sfard’s \(2009, p. 194\)](#) writings, gestures are “a bodily movement that fulfills a communicational function.” He also believes gestures and language are closely related. This finding is also defended by other researchers who focus on this theme. [Radford \(2020, p. 24\)](#) states that “language, signs, artifacts, and the body are also considered as part of thought. They are part of the material texture of the thought of individuals.” From the perspective of embodied cognition, [Edwards \(2003\)](#) considers that the connections between the gestures produced by people should be taken into account as a part that integrates language and thought.

We also point out [Vygotsky’s conception \(1997, p. 133\)](#) about gestures, where he states that “a gesture is specifically the visual sign [...] The gesture is a writing in the air, and the written sign is often simply a fixed gesture.” However, this gestural symbology does not refer only to something loose in the air drawn with the arms and hands; it carries meanings. Thus, we agree with [McNeill \(1992, p. 11\)](#) when he argues that “with gestures, each symbol is a complete expression of meaning in itself.” That is, a gesture can visually synthesize a specific mathematical concept in a single go.

Some consider that body language can be expressed through the look, facial expressions, gestures, and body positions.

This is the case of researcher [Neves \(2020\)](#). We emphasize that our understanding of gestures refers to the fact that they can be conceived as movements of one or more parts of the body, that is, they are externalizations through the body of a given individual. In this sense, [McNeill \(1992\)](#) mentions that gestures:

[...] are not just movements and can never be fully explained in purely kinetic terms. They are not just the swinging of arms in the air, but rather symbols that exhibit meanings in their own right. They have a meaning that the speaker freely designates. (p. 105)

As can be seen, the spontaneous movements examined in these studies are a subject of interest, and several of them discuss the relevance of gestures, as well as their meanings in the context of mathematics education.

That said, taking into account the various points of view of scholars on the definition of gestures that we have previously presented, we assume in this text that they can be conceived as a movement of one or more parts of the body and/or even facial expressions. In this sense, gestures are not limited to the use of these parts of the body. They have a broader meaning and can be perceived in facial expression and exchange of glances, as stated by [Xiong and Quek \(2006\)](#).

When discussing gestures, we are inevitably compelled to comment on their relationship with speech. Regarding gestures and the gradual emergence of speech in children, [Vygotsky \(2008\)](#) considers that:

Finally, in a group, the children do not play, and speech becomes the only mode of representation, with mime and gestural expressions disappearing. The



percentage of gestural actions in play decreases with age, while speech gradually comes to predominate. (p. 147)

As we have seen, over time, children reduce the number of gestures and, consequently, start to use speech more frequently and skillfully. However, although this is a natural action, we know that gestures do not disappear in adolescence, adulthood, and/or old age. They last until a few seconds before the person dies, since the body is still moving.

Gesture and speech: Which comes first?

There are two schools of thought about this duality. The first school states that gestures and speech do not always convey the same information. Church and Goldin-Meadow (1986) discussed the incompatibility between gestures and speech, particularly in instances where the gesture conveys new information not present in the discourse. In this way, gestures can reveal information that complements oral verbal language. The other strand considers that gesture and speech form a unified system; this occurs because there is usually a concomitance between them, i.e., speech is often accompanied by gestures. According to McNeill (2015), these two components form a single set. In this sense, during the act of performing a gesture, it will probably be necessary to use speech. Thus, for Silva

and Faria (2022), gesture and speech are elements that comprise language.

In this text, we consider cases in which gesture-speech synchronism occurred. Taking this choice into account, we clarify that gestures accompanied by speech are referred to as gesticulation or filling gestures, as described by McNeill (2000, 2006). The author also notes that these are instances of mandatory speech monitoring. Also, according to McNeill (1992), the gestures produced concomitantly with speech can be subdivided into four dimensions. We reinforce that the order established for this description does not imply a hierarchy for gestures. None of them can be considered the most important.

The dimensions of the gesture: the quartet

Initially, we noted that gestures were long categorized and/or typologized. Several texts and research bring this nomenclature. McNeill (2006, 2015) recommends using the term “dimensions”. According to the author, the arguments for this substitution of terminology stem from the fact that the meanings conveyed by the words “metaphorical,” “iconic,” “deictic,” and “rhythmic” refer to dimensionalities, and such dimensions can be present in a single gesture. This means that a metaphor can also fit into iconicity, just as in the latter, we can notice the presence of deixis. Chart 1 shows a synthesis of each of these dimensions.

Chart 1. *Synthesis of the dimensions of gestures*

Dimension	Dimension Feature
Iconic	They are closely related to the semantic content of speech. They visually represent the content of concrete entities and/or actions. They illustrate what is being said at the moment of speech (McNeill, 1992). A possible example might be when a person raises one of their arms with their palm extended downward and looks intently at that limb. Depending on the context, the individual is communicating the stature of someone present and/or absent at the time of the speech.



Dimension	Dimension Feature
Metaphorical	They are very similar to the iconic ones, but in this case, they can represent abstract content. These gestures can have a form and/or occupy a space (McNeill, 2006; Cavalcante <i>et. al.</i> , 2015). According to Costa (2010), abstractions take shape. For Rosa and Farsani (2021), an example can be the same case as the iconic ones, that is, a person raises one of their hands palm-down over their head with the intention of indicating “high intelligence.”
Deictics	They are characterized by the fact that the individual points to existing or virtual objects and actions in space, usually expressions such as: “here,” “there,” “above,” “below”, among other connectives. According to McNeill (2005), they can enable the materialization of abstract concepts.
Rhythmic	They are known for their simple, repetitive actions, which are used for emphasis. The hand performs a rhythmic function as if it were demarcating speech. A possible example occurs during the speeches of politicians.

Note: Prepared by the authors (2024).

In this text, we are not concerned with identifying the gestural dimension. We will focus on the meanings that gestures can bring, especially in contexts where discussions about geometry content. However, we are aware of the difficulties involved in constructing, understanding, and communicating geometric concepts that are spatial in nature. For example, the work of [Wagner *et al.* \(2004\)](#) concluded that participants in their research generally produced more gestures when building knowledge about spatial words, compared to moments when they did not need to resort to non-spatial concepts.

That said, we agree with [Skemp \(1993, p. 31\)](#) when he comments that “the communication of mathematical concepts is much more difficult, both for those who communicate and for those who receive the communication”. In this context, we introduce the abstract notions of Geometry, such as the concepts of *point*, *line*, *plane*, *semi-line*, *parallelism*, *triangle*, and *polygon*, among others ([Lima & Carvalho, 2010](#)). These abstractions are considered perfect and ideal in people’s minds, but their representations in the world are viewed as imperfect (Silva & Wagner, 2014). In view of these considerations, we are in line with the following statement, [Khatin-Zadeh, *et al.* \(2022\)](#):

When a mathematical concept or idea is metaphorically described in terms of gestures, embodied actions, or a fictitious movement, the motor system kicks in to substantiate and understand that concept or idea. This process of mapping, in which an abstract concept or idea is represented metaphorically as actions and body movements, can facilitate understanding. (p. 7)

[Neves’s \(2020\)](#) work aligns with this perspective. The author believes that a good example of the production of gestures is in dialogues where there are attempts to explain certain concepts considered more abstract. Following the same direction, [Jesus \(2021\)](#) describes situations during the production of data for his work in which participants had to be challenged to solve geometric problems considered too abstract. According to the author, individuals began to mobilize their bodies so that the concepts and/or properties of the mathematical object discussed at the time took on meaning. This posture enabled a better understanding, as well as made mathematical notions more palpable both for those who produce the gesture and for those who are present in mathematical communication. In view of



the above, we agree with the ideas of [Chen and Herbst \(2013\)](#):

It is notorious that [...] when students present their conjectures, the use of gestures helps them to develop and communicate complex explanations without the need to use formal mathematical language; Thus, gestures can allow students to participate in discussions about concepts before all of these have been formalized and represented in formal language. With gestural and verbal expressions, students can communicate more of their reasoning and thinking to their classmates and teachers. (p. 286)

Thus, the production of gestures in contexts of knowledge construction, especially when it comes to concepts loaded with abstractions, can promote access to geometric knowledge to a greater number of students. In the words of authors [Khatin-Zadeh et al. \(2024, p. 5\)](#), the use of gestures is important, as they help “students acquire a grounded understanding of mathematical concepts”. To illustrate the production of gestures by students, we present a series of situations that address geometry concepts. For example, the work of [Elia et al. \(2014\)](#) indicates a case of iconic gesture involving five-year-olds. For these authors,

[...] When the child gestured about the shape of the various blocks to which he referred in his description, these gestures were of an iconic character. An example concerns the cylindrical shape (lines 46-48), to which the child moved his finger to make a round line vertically in the air. (p. 746)

The previous example brings a sign represented by the child through his body.

There was an intention in carrying out such a gesture. Initially, we can understand that this participant can probably externalize the characteristics of a cylinder (a geometric solid). It often happens that children in the initial phase of schooling have not yet appropriated the geometric vocabulary. For example, the following situation may occur ([Silva, 2018](#)):

When a student refers to the defining attributes of geometric figures such as points, scratches, etc., or is unable to express themselves, resorting to gestures and drawings, they demonstrate that there is still a need for further development of verbal ability and vocabulary proper to geometry. (p. 27)

On the other hand, the wealth of gestures used to express mathematical concepts underscores the relevance of our body as a primary means of communicating mathematical thinking. According to [Eli et al. \(2014\)](#):

[...] different aspects of the geometric content were more likely to stimulate the child’s use of specific types of gestures. When the child described the shape (e.g., a cylinder), the orientation of a block (e.g., a horizontal direction), and topological relations of proximity or separation (e.g., attached or unattached shapes), he tended to produce iconic gestures that represented the geometric aspects involved. (p. 755)

Drawing on the ideas presented in [Elia et al.’s \(2014, p. 756\)](#) work, we arrive at the following conclusion: “In particular, the results of this case study suggest that the iconic gestures were more appropriate and more pertinent to the child’s geometric thinking than the deictic gestures.” Although the



cited work focuses on children, it is also relevant to conduct investigations involving adult participants in contexts where they produce written signs in the air. An issue to be observed concerns the great use of iconic gestures to build geometry notions. This type of dimension can present the illustration of the shape of mathematical objects in the air as a characteristic, as geometric concepts require representations through images, drawings, and other means. Individuals often resort to outlining specific geometric figures in the air.

Neves's (2020) doctoral thesis is a Brazilian example of research in which the author concluded that participants performed gestures to indicate the shape of objects being discussed. The work by Jesus (2021) also dedicated one of the categories of his investigation to the gesticulations produced by the study's collaborators. One of the results shows that:

Often, in solving the various questions, we observed undergraduates pointing out where the colleague who wrote should write down an expression or in which word or other graphic representation of the question the information to which they referred resided. In general, these gestures are accompanied by expressions such as "here," "there," "this, look," "from here to here," "this here," etc. (p. 217)

These expressions are characteristic of deictic gestures that have the function of pointing. This type of gesture was also performed by prospective mathematics teachers who participated in Scheffer's (2006, p. 186) research. The author states that "the movement of the hand, the gesture of pointing with the index finger along the graphs

were present in the students' explanation and interpretation of the curves representing body movements in the Cartesian plane." There are many possibilities for the production of the deictic gesture. Neves (2020, p. 202) observed that "students also use deictic gestures, through a laser penlight, to define and highlight ideas related to the concept of circumference using the Ferris wheel and a circumference in the video." The purpose of the problem was to discuss the displacement of a cart when it made a complete turn.

The deictic gestures are crucial in human communication. They have been present since our childhood. Some peculiarities of this dimension were commented on by Vygotsky (2008, p. 37). This theorist states that "Stern himself admits the mediating role of gestures, especially that of pointing, in establishing the meaning of the first words. The inescapable conclusion would be that the gesture of pointing is, in fact, a precursor to the intentional tendency." For the author, at first, this gesture may indicate a desire on the part of the child to take something; however, they are not successful in doing so. They apply several unsuccessful strategies, for example, their fingers perform movements with the intention of picking up objects that are nearby (Franco & Martins, 2020):

We start from the assertion that it is not the hand that writes, but the thought. Writing is about expressing ideas, and for this, the child needs to be able to abstract the sensory aspects of speech and advance in the construction of a language that does not use the oral word, but its representation. (p. 122)

That said, we understand that gestures produced in the context of discussing



mathematical concepts can be produced intentionally. These movements constantly influence our thinking.

Edwards (2003) discusses an episode reported by McNeill (1992) regarding the act of producing gestures for specific mathematical concepts. Some questions can be formulated within this theme. For example, a) Can the same mathematical concept be represented by means of the same gesture by different people? b) Does the same individual produce similar gestures at different times for the same mathematical concept? On this subject, let us look at what McNeill did (Edwards, 2003):

McNeill analyzed a videotape of two mathematicians in conversation and determined that, unlike other spontaneous gestures, the hand movements that corresponded to various mathematical concepts were similar between the two speakers. In addition, the same gestures were used for the same concepts at different times in the conversation, and a given gesture seemed to correspond to a single concept (rather than representing a complex of concepts or events. (p. 4)

Following this direction, we will seek to reflect in this text on the situations in which students discussed the primitive entities, *point*, *straight line*, and *plane*. The gestures produced to communicate these notions were observed among students of the same pair, between different pairs, and, finally, by comparing the gesticulations performed in the first meeting with each pair of students with those produced in the second moment. In this scenario, did each student produce the same gesture to represent the idea of a *plane*?

In light of the theoretical repertoire exposed here, we present three situations that

mobilized students to produce gestures to construct, communicate, and/or explain concepts and/or notions within the field of SGP. They concern the elementary notions of geometry, namely, *point*, *line*, and *plane*, which are primitive entities communicated through different gestures; however, they show the interconnection between thought, body, and speech.

Materials and Methods

In this section, we present the methodological approach for collecting data from the characters involved in our study. We began with our conception of qualitative research, then focused on describing the research context, including the process of conducting interviews with prospective mathematics teachers. We also discussed the methodological procedures, as well as the resources used and the reasons behind these choices. Finally, we recorded the analysis process adopted.

Considerations from the qualitative perspective of research

The interpretative nature predominates in this research. Qualitative bias was not chosen randomly. It is present in this work due to the objective we have been pursuing since the process of elaborating the research project. We value the processes, the voices of the participants, and the different forms of communication used by the characters in our study.

In this sense, during the production of our data, we continually questioned this mode and the strategies used by participants to express their understandings of the content related to the SGP discussed between students and researchers.



Situating the research that gave rise to this text

This text is part of a broader study that collected data from prospective mathematics teachers at a Brazilian public university in the state of São Paulo. The research was developed in two central axes. In the first axis, we had two meetings with a group of students enrolled in the Spatial Euclidean Geometry subject. In total, there were 32 participants, with ages ranging from 19 to 23 years old. In the second moment, referred to as a teaching experiment in light of [Steffe and Thompson's \(2000\)](#) ideas, we conducted interviews with six pairs of undergraduate students from that universe, who were chosen based on previously established criteria.

Specifically for this text, we analyzed the interactions between pairs of students during the second moment in October 2022. Each pair was interviewed twice by the first author of this text, with an interval of approximately seven days between interviews. The interviews lasted an average of 90 minutes and were recorded on video. In them, the questions asked were thought out, elaborated, and discussed in plenary by the members of the research group Teorema – Interlocutions Between Research and Mathematics Education. They dealt with SGP concepts, aiming to promote discussions that would enable researchers to analyze how students understand and express these notions in the field of geometry. The collaborators were assigned fictitious names to have their identities preserved. They also signed the Informed Consent Form, in which some chose not to reveal their identities. The research is registered under number 5.278.828 with the Human Research Ethics Committee.

From the various elements that comprise the curriculum of Spatial Euclidean Geometry discussed between the participants and the researcher, we have decided to address in this manuscript the three initial elements, known as primitive entities, namely: *point*, *line*, and *plane*. They were constantly mentioned by the pairs, as well as the production of gestures to represent and communicate them in the most visual way possible. In the latter case, we sought to understand the meanings of these gestures and, at the same time, to verify whether they maintain a pattern, that is, which parts of the body and how the students produced the gestures.

About the questions that promote discussions of geometric concepts

During the data collection phase of the research, we utilized a questionnaire with open-ended questions that addressed SGP issues. In each meeting, ten questions were discussed that were posed to the pair spontaneously; i.e., we did not read them beforehand. Instead, we promoted a conversation with the students. They aimed to promote discussions about specific concepts from the SGP, while also revealing how students express and understand these ideas in the area of mathematics. For this text, we delimit the dialogues that occurred in the moments when the collaborators resorted to primitive entities to justify their speeches during expositions of postulates, theorems, and mathematical propositions. Thus, the corpus of analysis will be based on this mathematical context.

Strategies and methodological resources

We emphasize that the research did not aim to conduct a study on gestures in the context of mathematics education. This



theme emerged from the interpretations made in the data produced. However, as we used high-quality audiovisual recording instruments and carefully positioned them to capture the students' movements, it was possible to perceive the production of gestures by these characters. In Figure 1, we provide an illustration showing the proper placement of the pair of students and the researcher, as well as the digital resources used. Next to it, we include a photograph of the location where the teaching experiment took place.



Figure 1. *Illustration of characters and digital resources*

Note: Authors' Archive (2024).

In the previous image, we can see an illustration of two “dolls” representing the pair of students, with a third indicating the figure of the researcher. Under the table, there are various items, including a cell phone, a laptop computer, and a camcorder, as well as an area reserved for making manipulable materials available to students.

Regarding the professional camcorder, we chose it because we believe it contributes to the data analysis process, to the extent that, when well-positioned, it can provide researchers with access to the most expressive behaviors of the students during discussions of the geometric knowledge communicated and constructed among those present in the dialogue. In addition to

the images provided, these technological devices can produce good-quality audio. Such a scenario contributes to the transcription process, which will be commented on later. The laptop computer and the cell phone played supporting roles. The first technological resource was used to capture the audio of the dialogues that took place between the students and the researcher. Although the camera also fulfilled this function, we opted for a second means of capture as a way to ensure the recording of data, thereby avoiding the need to return to the natural environment due to a possible technological failure. The second resource recorded the images and the audio. It was leaning on a tripod, focused on the pair of students. In total, there were approximately 19 hours of recordings from the 12 meetings, with two recordings from each of the six pairs of collaborators.

The audiovisual material was fully transcribed. For the success of this process, we have benefited from the assistance of software. For example, *Sonix* and *Descript* (<https://sonix.ai/pt> and <https://web.descript.com/>) offer free but limited versions. In our case, we chose to purchase a monthly account of the second software until the completion of the transcriptions. These were separated by the date on which the meeting took place, as well as the names of the members in each pair. We read through the material several times. This procedure was essential for us to analyze the excerpts that occurred in the gesture-speech synchronism. As mentioned, for this text, we analyzed the transcripts that referenced the primitive entities *point*, *line*, and *plane*.

The spider's web: our reviews

We are considering the process of analysis as a spider's web, a metaphor brought by Lemann that symbolizes a



cohesive connection between the elements of an organization. The spider weaves its web in an efficient, elegant, and valuable way for its survival. To analyze our data, we must make connections between the different types of information produced, whether communicated through drawings, oral and/or written verbal language, or body language. From there, we must make a bridge with the theoretical framework we have chosen. Thus, the analyses we have made about gestures to communicate primitive entities are considered a great spider's web.

That said, these analyses took place through the triangulation of methods and data. The primary source of the data was the transcripts resulting from conversations between students and between the researcher and the students. The videos of the interviews were watched numerous times with the intention of observing the regularities, repetitions of patterns, behaviors, and various movements. As we returned to the recordings, we noticed new elements. From then on, we started taking notes. This process was crucial for us to detect the production of gestures that represent certain notions of the SGP, among them the primitive entities *point*, *line*, and *plane*, which are mathematical objects of discussion in this text. When we observe these gesticulations, we consider it relevant to record them. To achieve this, we created a file containing the *frames*. A frame contains an image of the exact moment when the student produced the gesture. They also composed one more item to be analyzed in depth.

We analyzed verbal, oral, and written languages along with the exact moment the gestures were produced. As already mentioned, the frames served as a means to display the gestures. Nevertheless, since the characteristic of an image is its absence of

dynamicity, we also chose to present the gestures using QR codes, which link the reader to short videos that show students' gestures to convey the concepts of *point*, *line*, and *plane*. This material was obtained through the identification of gestures in numerous instances of watching the recordings. The next step was to make cuts in the video to identify the time interval during which gestural production occurred.

After watching the short videos countless times, we grouped them into specific folders stored on the computer, separating them by the geometric concept to which the gesture referred. In the next step, we carefully examined the gestures produced to understand their meaning and verify whether the way in which a specific gesture occurred for a particular student also prevailed for other participants in other pairs. In addition to these elements, we attempted to observe the presence and/or absence of gesture-speech synchrony. Thus, the purposes of our analysis were to examine the variety of gestures associated with the participants' discourse regarding primitive beings.

In the following section, we will discuss our analyses in the light of the theoretical framework.

Presentation and discussion of results

The purpose of this section is to present the data produced with the pairs of students during the second stage of the research. At the same time, we will present these data and analyze them in the light of the chosen theoretical framework. We aim to illustrate the meanings of these gestures in the mathematics field. The section will draw from three streams, each alluding to an element. For example, in the first case, we illustrate



the gestures produced whose intention was to convey the idea of a *point*. In this case, we observed the students during moments when they resorted to this geometric object to provide some explanation. In the second case, we deal with body movements intended to represent the notion of *line*. Finally, in the third, we present excerpts from cuts that refer to the *plane*.

The first current will be represented by a discussion that took place in an experiment involving sheets of paper. Our purpose was to motivate students to perform folds in such a way that they formed right angles. The purpose of the activity was to show through manipulable materials the fundamental theorem of perpendicularity. That said, at a certain point, the researcher asked: “*So, for a line to be perpendicular to the plane, what is needed?*” to which student Cleonice answered: “*Let it be perpendicular to all the lines belonging to this given plane.*” The researcher continued: “*But is there a way for me to check this for all the lines of the plane?*” The other

member of the duo, Renata, said: “*I don’t think so.*” Cleonice insisted by saying: “*There is, we did, didn’t we? The demonstration.*” The interviewer continued: “*For all the lines of the plane?*” At that moment, student Cleonice began to produce three gestures, one after the other, each of which referred to a primitive entity of geometry. She said: “*Take the intersection point between the line and the plane.*” The first underlined expression alluded to the notion of the primitive being *point*. The following two indicated the elements *line* and *plane*. The first image in Chart 1 depicts the gesture produced by an index finger resting on the table, pointing to the intersection of the two edges of a paper, each of which represents a straight line that intersects at the place indicated by Cleonice. In the second image, the student raised one of her index fingers, and her other hand, palm down, touched that finger.

Through the video available via the QR Code, it is possible to see the gesture and speech synchrony, that is, as Cleonice spoke, she produced the corresponding gesture referring to her speech. We acknowledge the presence of McNeill (1992), who exposes the fact that gesture is an integral part of discourse. This scenario can also be confirmed by the fact that the student pronounced the word, alluding to a *point*, and then verbalized the second element, *line*, representing it by another type of gesture. Finally, the

Chart 2. Triple gesture produced by student Renata to indicate the ideas of point, line, and plane



Note: Authors’ archive.



notion of *plane* was also represented by another movement that was consistent with what Cleonice pronounced. The triple gesture for each elementary notion of geometry produced by the student demonstrates what McNeill (1992) defends, i.e., that each gesture has a meaning in itself. In the case we presented, the index finger pointing to the meeting of three creases on the sheet of paper represented the same *point*. The same finger raised upwards indicated the idea of a straight line. The palm of one hand facing down represented some a plane.

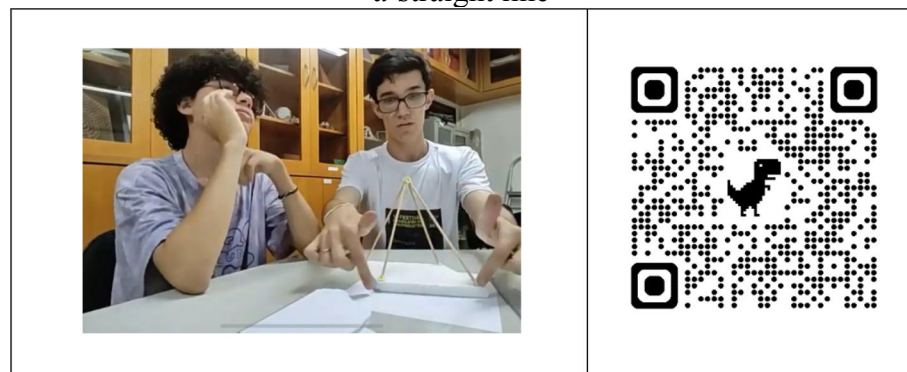
The second school of thought focuses on the notion of a straight line. The debate was promoted through a question accompanied by a pyramid representation. At the time, the first author of this work said: “*I brought a representation of a pyramid in which the base ABCD is a parallelogram. What can you say about the CD edge and the plane determined by the VAB face?*” The purpose of this question was to discuss one of the theorems of Euclidean spatial geometry, namely: “Let β be a plane and r a line not contained in β . So r and β are parallel if and only if there is a line s contained in β and parallel to r .” Soon after the researcher concluded his considerations, student José began presenting his arguments. He said: “*This edge is parallel to the plane.*” The researcher wanted to know more details about the student’s justification for this statement. José replied: “*Because the plane contains a line that is AB that is parallel [...]*” The primitive line was introduced

by the student at the same time he produced a gesture with his two index fingers, which touched each other at a particular moment, sliding in opposite directions over the edge of a piece of Styrofoam.

José’s gestures to communicate the notion of a *straight line* show what Radford (2020) has been saying about the fact that the body is part of thought. We agree with the author because the student was able to explain the parallelism between a *straight line* and a *plane*. In our understanding, there was a common thread between thought and body language, one helping the other in the process of argumentative exposition. In Chart 2, the exact moment of gesture production is shown.

The fact that José made his index fingers touch each other and, at the same time, made movements in opposite directions suggests that he wanted to draw attention to the infinity of the line. As already mentioned, this movement took place on the edge of a piece of Styrofoam. The intention of sliding over this material may indicate that the student wanted to emphasize the characteristic of a straight line. Finally, José managed, through the use of manipulative materials and gestures, to visually demonstrate that the parallelogram contains a *straight line*

Chart 3. A gesture produced by the student José to indicate the idea of a straight line



Note: Authors’ archive.



parallel to another that is contained in the *VAB plan* (one of the faces of the pyramid representation). Hence, the edge *CD* is parallel to the above *plane*. The student appropriated speech and gestures to ensure that the communication of his reasoning was as clear as possible to those present in the discourse, as defended by [Chen and Herbst \(2013\)](#). These mathematical objects were represented by means of gesticulations; In addition, the student, while producing the gestures, looked attentively at them.

We could also perceive another potential during this communication and/or the construction of abstract notions through gestures in student Edson's behavior. When José said, "*This edge is parallel to the plane*," his colleague looked at the gesture produced, and, after the end of the verbalization of the sentence, Edson produced another gesture with his head, nodding as if he wanted to agree with what José said. To conclude, the student made another gesture by exchanging glances with the first author of this text. These observations confirm what is defended by the main gesturologists, namely, that gestures also aid in discourse and are produced for the other, as with Edson. This, in turn, produces more gestures, continuing the representation of mathematical concepts through gesticulations. In summary, the nodding of the head, implying agreement with the colleague's argument, as well as the exchanges of glances between the pair of students and the researcher, suggest that these movements can also be considered gestures.

In the successive two frames, we show the idea of a plane being represented through gestures by two different students and members of different pairs. The two situations occurred on different days, and the discussions originated from non-identical questions. In Chart 3, the gesture produced

by José occurred as his two hands moved in opposite directions, with their palms facing down and loose in the air, as if they were sliding across the table. These actions were carried out based on the following question: *How do you understand the notion of perpendicularism in the plane?* Several explanations emerged, for example: "*It has to do with two lines that are perpendicular*" (Edson), "*perpendicularism; a good idea that there are similar figures*" (Edson). This student at one point made a long comment about perpendicular lines:

For me, this is a difficult question to answer, at least for me, because, when we are in the abstraction of geometry, it is an ideal world, a 90-degree angle, it is half of a shallow angle, you know, it is exact; In the real world, we can't measure this so accurately, we don't know what a 90-degree angle is in real life. But, by approximation, an angle that approaches 90 degrees is a 90-degree angle, a right angle, and then, if these lines define an opening of 90 degrees between them, we say that they are perpendicular (Edson's comment).

In Edson's speech, several passages promote good geometric discussions. Initially, the student recognizes the difficulty in articulating his position regarding the concept of perpendicularism in a plane. Then, he comments on the fact that right angles are not found in real life; these are concepts in the world of ideas. Edson believes that what we can find on a daily basis is an approximation of reality. In the student's comments, there is also the notion of angle, understood as an opening, and when it tends to 90 degrees, the lines are called perpendicular.



Following the richness of details presented by José, we invited the pair of students to experiment with sulfite sheets, aiming to produce perpendicular lines. After a few conversations, we spontaneously explored the concept of perpendicularism in space. It was on this occasion that the gesture to indicate the idea of a plan took place. We observed José's report:

I think that given any plane, not necessarily just a line, but any set of lines, if the intersection of it with the base plane, which we use as a reference, it establishes an angle of 90 degrees, [...], then we will have perpendicularism; and, if they meet, depending on what the position is in that plane.

José's speech, as exposed above, referred to the fundamental theorem of perpendicularism; however, the cut that interests us at this moment is the one that is underlined. At the end of his explanation comes the notion of a *plane* being indicated by gestures. Gesticulation was performed concomitantly with speech, as can be seen in the video available via the QR code below.

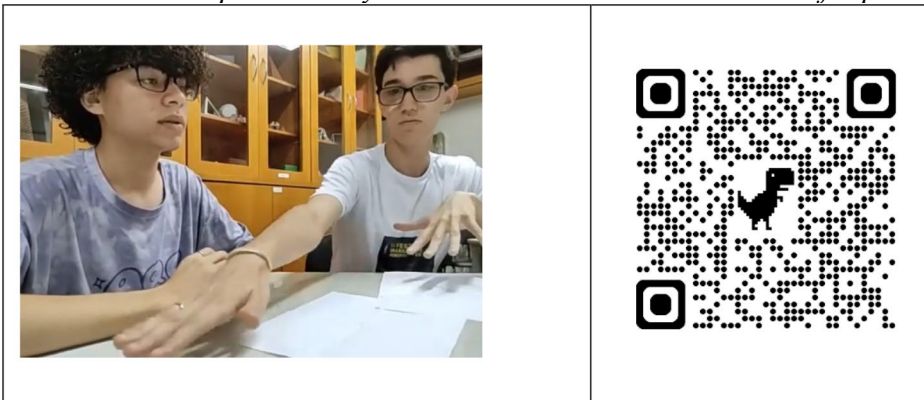
The gesture in frame 3 indicated the idea of a plan. By leaving his hands with

his palms down and loose in the air, José implies intentionality. Initially, he looks closely at the gesticulation he produces. His hands sliding in opposite directions and suspended in the air may indicate the infinity of the plane. This type of gesture, used to represent one of the primitive beings, a *plane*, by means of the palm facing downwards, was also produced by Renata, as shown in Chart 1. The gesto-*plane* shown in this way provides indications of the need for a more detailed investigation into the meaning of this gesturing, as other students have also appropriated this resource to communicate what is meant by *plane*. This is the case of student Leonardo, presented in Chart 4.

In the first image shown in frame 4, there is a mobilization of the entire body of the student Leonardo to show how the representation of a *plan*. This gesturizing was produced in response to the same question that generated the discussion in Chart 2, i.e., in the debate around the idea of *line* represented through gestures by the student José. To reach the moment of gesticulation, the student explained that the *straight line* will never intersect with the *plane*. This argument of his was valid, given that we were discussing the theorem of a *straight line* parallel to a *plane*. The student said, "Because they'll

never meet. The straight line will never meet the plane. Because the plan does it like this, right? [...] it comes to and fro [...] and the straight line will never meet this plane." Leonardo joins his two hands,

Chart 4. Gesture produced by student José to indicate the idea of a plane



Note: Authors' archive



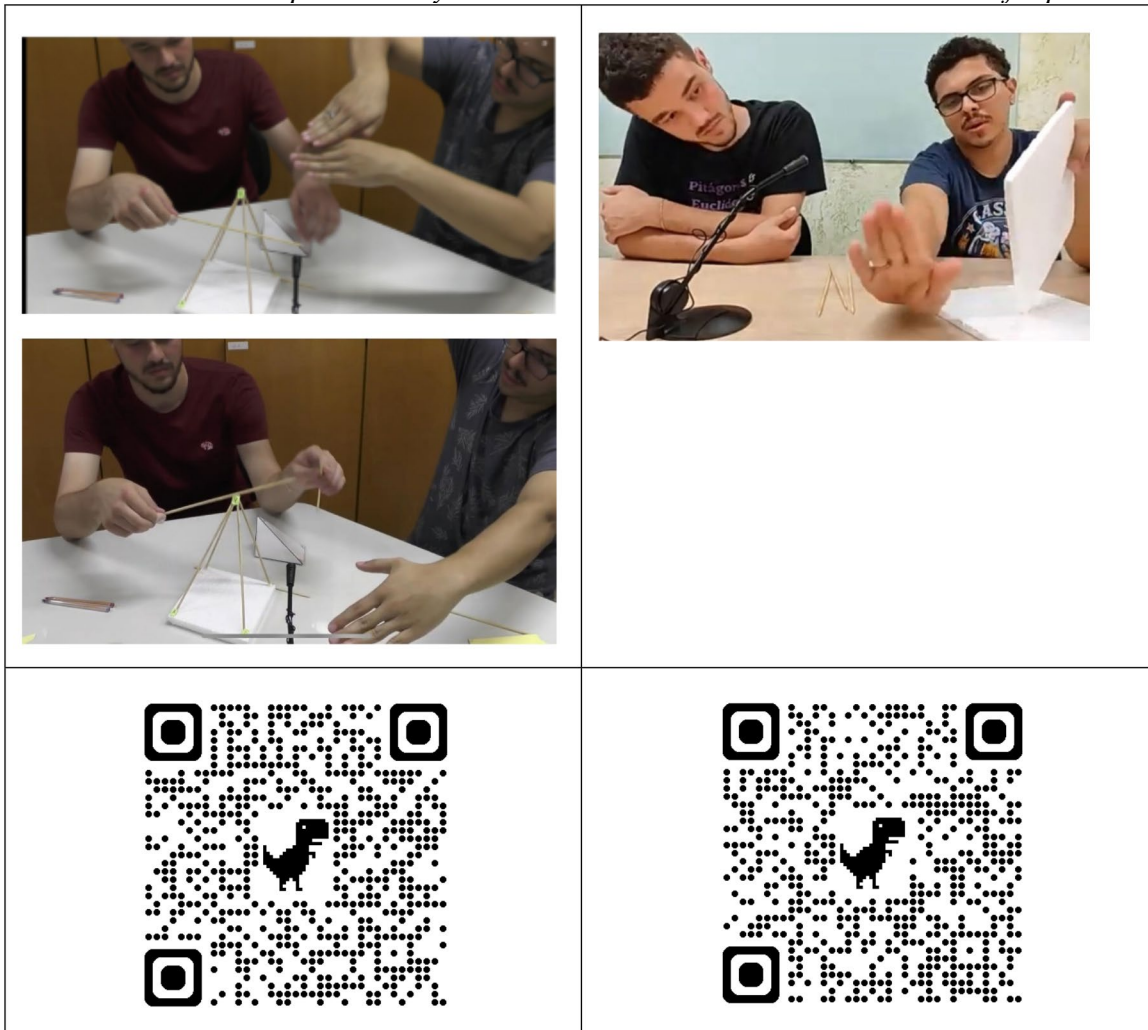
looking at them, and then performs inclined movements in opposite directions. These attitudes show that the student can externalize the infinity of the plane through his body, as its expansion to all sides has been made clear. These statements are evident in the first two images in Chart 4.

On another occasion, Leonardo performed a gesture similar to the previous one (second image in Chart 4). The intention was also to represent the idea of a plan. The eyes of the two students followed Leonardo's hand movements attentively. They

were talking about the intersection between two planes. Thus, we (the first author) asked: *"Will the intersection of these two planes be a single point?"* Leonardo said:

It is limiting. Only, like, here, it's going to be infinite. So it goes away here, goes away here, goes away here, goes away here. So in that case, yes, the only possibility is going to be a straight line, which in this case here, is a piece of a plane. But the plane continued here, it continues here. (Leonardo's report).

Chart 5. *Gestures produced by the student Leonardo to indicate the idea of a plane*



Note: Authors' archive.



The expansion of the *plane* in the four senses can be identified through the use of the expression, “*goes away here*,” which Leonardo uttered four times, simultaneously with gestures indicating different directions. In addition to the gestures produced with the hand and the looks, a third gesticulation occurred when the student was finishing his argument. He raised his hand to the height of his chin and smoothed it, at the same time looking carefully at the manipulable material (Styrofoam) that played a preponderant role in both gesticulation and promoting visuality. We understand this last bodily mobilization as if it were the *thesis* of a theorem, since the student was concluding his reasoning about the fact that the intersection between two *planes* is necessarily a *straight line*. Leonardo said: “*So, in that case, yes, the only possibility is going to be a straight line*.” In this way, the quartet formed by gesture, speech, visualization, and manipulative material worked together to externalize the idea of a *plane*, and not only that, but also constructed and understood an important property of the SGP: in Euclidean geometry, the intersection between two *planes* cannot be just a *point*.

The gestures used to externalize the idea of a *plane* involved moving the palms of the hands downwards, with constant movements that often involved one or two hands. In the latter case, they moved away from each other to indicate the infinity of the *plane*. These findings are in accordance with [Edwards \(2003\)](#), who presents the results of a study by McNeill, namely, that identical gestures are produced for the same concepts. For the case of our study, gesticulation to indicate the idea of a *plane* was carried out by different students on different days and/or at the same meeting.

Final considerations

The preliminary notions of a *point* and a *line*, as discussed in Charts 1 and 2, were represented by the students by tracing their fingers on any surface. In the case of the one-dimensional entity, a deictic gesture was present, indicating a *point*. In this context, it indicated an abstract object, suggesting that deixis facilitates the process of visualizing a notion present in the realm of ideas.

The situations presented in this text demonstrate that the students participating in the study utilized gestures as a means to explain their thoughts during discussions of geometric properties. We have noticed that when subjects do not have words to express their thoughts about a significant mathematical result, they often represent it through gestures. A possible example was the exposition on the theorem of a *straight line* parallel to a *plane*. In this context, the pair of students who approached this theme through gestures demonstrated that the *straight line* positioned opposite one of the faces of a pyramid representation was parallel to the *plane* determined by said face.

From the cases presented in this text, we reveal that in all of them, gestures were produced together with speech, that is, they are integral parts of discourse. This confirms the studies of several researchers who have investigated the production of gestures by both mathematics students and teachers. Thus, we understand that mathematics and the body are two complementary aspects that fit together. In other words, they are not antagonistic; instead, they are complementary and can be important in the process of constructing mathematical concepts.

We agree with [Hostetter and Alibali \(2019\)](#) because, according to them, when gestures occur in conjunction with speech,



they reflect our thinking, especially thoughts related to spatial notions. In the case of primitive entities, the notion of a *plane*, a two-dimensional mathematical object, was represented in a three-dimensional Euclidean space by means of two hands open and loose in the air. This idea was freely drawn by some students as movements simulating the characteristics of the object, and, in parallel with the gesture, speech occurred, indicating which directions and/or movements the hands should follow.

In short, another result already presented in the theoretical framework concerns the fact that students produce identical gestures (open hands with palms down and/or up) to represent the same mathematical object (plane). These actions were carried out by the same students on different days; however, there were also instances where other individuals produced gestures. These considerations are relevant and demonstrate the power of gestures, as they can provide clues to how students express and/or learn concepts of greater complexity.

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We declare that this research was approved by the Human Research Ethics Committee under process number: 5,278,828. The participants read and signed the Free and Informed Consent Form.

Conflict of Interest

-The authors declare no competing interests.

Author contribution statement

All authors declare that they have read and approved the final version of this paper.

A.F.L.: Validation, Investigation, Re-sources, Writing - Original Draft. R.B.A and D.F.: Original Draft, Writing - Review & Editing. A.F.L; R.B.A and D.F: Validation, Investigation, Resources, Writing - Original Draft.

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Data availability statement

The data supporting the results of this study will be made available by the corresponding author, A.F.L, upon reasonable request.

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