# MODELING VOTING DYNAMICS IN A TWO-PARTY SYSTEM

# MODELAR LA DINÁMICA DE VOTACIÓN EN UN SISTEMA DE DOS PARTIDOS

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## Abstract

The 2016 U.S. presidential primary election, characterized by unexpected results, provides an interesting context to study how citizens are influenced in deciding whether to vote and whom to support. Our aim is to determine which of those changes in voting behavior have the largest impact on the election outcome. We address this question by developing a class of models driven either by the effect of mass media or by social interaction among voters and non-voting members of two parties. The dynamics are modeled using four compartments with a transition matrix describing the evolution of a discrete-time Markov chain. Each model is studied and fit to poll data from the 2012 and 2016 U.S. presidential elections using numerical methods. A comparison across elections indicates that the social influence of each group changes from one election to another, but response to media is similar in both cases.

**Keywords:** Markov chain; dynamical system; voting behavior; peer influence; media influence.

## Resumen

Las elecciones presidenciales de 2016 en los E.E.U.U., caracterizadas por resultados inesperados, proveen un contexto interesante para estudiar cómo son influenciados los ciudadanos en su decisión de voto. Nuestro objetivo es determinar qué cambios en la intención de voto tienen mayor impacto en el resultado. Con este fin desarrollamos una clase de modelos basados en o (1) los medios de comunicación masivos o (2) las interacciones entre los votantes y miembros de dos partidos. Dividimos a la población en cuatro compartimientos con una matriz de transición que describe la evolución de una cadena de Markov en tiempo discreto. Estudiamos cualitativamente cada modelo y utilizamos métodos numéricos para ajustar los parámetros a datos provenientes de encuestas previas a las elecciones de 2012 y 2016. Encontramos que la influencia social en cada grupo cambia de una elección a la siguiente, pero la respuesta a los medios de comunicación es similar en ambos casos.

**Palabras clave:** cadena de Markov; comportamiento electoral; influencia de pares; influencia de medios de comunicación.

**Mathematics Subject Classification:** 91F10.

# 1   Introduction

The 2016 presidential election was characterized by several unexpected events. Notably, Hillary Clinton lost the presidential race to Donald Trump by electoral vote, despite gathering more votes nationwide by the largest margin in United States (U.S.) history [2, 9]. Trump's rise within his party was itself unique: not only did he receive the most media coverage for the least cost in the race [18, 64], but he also achieved the most primary votes of any Republican in history, exceeding Governor George W. Bush in 2000. He also received the most votes against a GOP candidate in history [7].

The apparent uniqueness of the 2016 primary race in terms of amount of media coverage and polarization of opinions motivated us to analyze the changes in political preferences among the U.S. population during the months previous to the election in comparison to the previous 2012 election. The aim of this study is to determine, in both the 2016 and 2012 elections, which changes in voting behavior (specifically, voting vs. not voting and choosing a party) had the most impact on the election outcome (regarded here as popular, not electoral, vote).

A well-known mathematical approach for studying elections and voting behavior is the "Michigan-style approach" [48], where analysis on the behavior of voters is conducted using poll data. Additionally, factors that influence groups of voters to make decisions, such as social interaction, media consumption and the economy, have also been studied extensively [15, 48, 28, 31].

Since person-to-person interactions and media coverage are two of the main ways in which the "campaign effect" of a candidate reach people [48], we limit our focus to these two mechanisms and use poll data as our measure of voting intention. In our two-party system, both candidates are competing for votes through the spread of an idea [3]: *"it's in your best interest to vote for me"*. Thus, we can view voting as an epidemiological phenomenon, in the sense that individuals can *infect* others to act as they do, i.e., voting is *contagious* [33]. This approach has been applied to many areas outside of epidemiology, such as fanatic behavior [10] and, directly relevant to our discussion, political parties [21, 40].

Person-to-person interactions can act as social pressure to push nonvoters to vote [24]. In fact, in election campaigns, the goal is not necessarily to change individuals' minds, but to get them to vote [14, 16]. Moreover, social contacts are more likely to drive the participants' opinions closer together than further apart, as observed in [24]. Agent-based models of copying opinions can be found in [15, 58]. In our model, we account for this effect and its opposite, known as the "backfire effect": interaction with an extremely polarized population can drive one subject's opinion drastically away from the other's view [34, 49].

This implementation is motivated by the "divide-and-conquer" strategy adopted by Trump, who benefited from emphasizing the divisions inside his own party [45].

While social interaction and word-of-mouth recruitment have been shown to account successfully for the growth of political parties in some contexts (see [21]), we hypothesize that mass media influence might be the driving force in the 2016 U.S. election in particular. One of the suggested methods for modeling media influence on voters' behaviors is TV exposure (see Zaller in [48]), which we use in combination with the TV mentions-by-candidate database in [77] and coverage studies in [64, 65]. Zaller proposes several models which determine voting preference of individuals at a given time as a function of media exposure at that moment. We explore a similar approach, in which an average individual changes his/her voting position in response to current media output. Moreover, we consider distinctly the effect of positive and negative coverage for a candidate.

In the next section, we formulate our models and notation, state our assumptions for each model and introduce our methods for data fitting and sensitivity analysis. We then display and analyze our results in their corresponding political context, in order to answer the research question. Limitations of our approach and suggestions for possible future work are given in the closing section.

## 2    Methods

### 2.1    Model formulation framework

Even though the effect of a third party may be critical under some conditions (for example the presidential election in 2000, see [40]), we focus on fluctuations between the two major American political parties. Thus, we consider a two-party system for our model, e.g. a Democratic-Republican system, and a population in which every individual belongs to one party.

The considered population is divided into four classes:

- $V_1(t)$ – Voters for Rep. candidate.
- $V_2(t)$ – Voters for Dem. candidate.
- $M_1(t)$ – Non-voting Rep. members.
- $M_2(t)$ – Non-voting Dem. members.

$V_1(t)$ and $V_2(t)$ represent the total number of people within the two parties who report themselves to be voting for the Republican or Democratic candidate at time t, respectively. The classification of $V_1$ and $V_2$ is specific to the candidate of each party. However, we do not directly examine the individuality of each candidate, e.g., their behaviors, credibility or ideology.

Instead, we use our models and empirical results to ascertain the most important movement(s) within the population in each election. $M_1(t)$ and $M_2(t)$ count people who self-identified with the Republican and Democratic party but have no intentions of voting for either party at time $t$, respectively. For the rest of the paper, we will refer to these categories as $V_1$, $V_2$, $M_1$, and $M_2$ for the purpose of reducing clutter in equations, time being implicit. The total population, $N$, only consists of people from the two parties, so $N = V_1 + V_2 + M_1 + M_2$ and $\frac{dN}{dt} = 0$. We use an epidemiological approach to characterize the different interactions and population transition between each group. Similar applications have been done recently in [40] and [43]. We treat influence similar to infectivity in the sense that people in some groups can drive other people to change class. However, influence can both attract and repulse others, whereas infectious individuals usually attract people from different classes to their own group (or a similar one).

We consider six types of possible transitions in one time-step, as in Figure 1:

- A non-voting member becomes a voter of the same party, $M_i \to V_i$.

- A non-voting member becomes a voter of the other party, $M_i \to V_j$.

- A non-voting member switches to the other party, $M_i \to M_j$.

- A voter becomes a non-voting member of the same party, $V_i \to M_i$.

- A voter switches side to vote for the other candidate, $V_i \to V_j$.

- An individual stays in its group, $V_i \to V_i$ or $M_i \to M_i$.

Notice that there is no direct transition between a voting class and the opposite member class. This comes from the assumption that, if committed voters change their candidate preference, they will become committed voters for the other candidate ($V$), not merely sympathetic ($M$). This is supported by extending the conclusion that once a person votes, it increases the chance of that person voting in the future, so once a person intends to vote, it is likely that, that person will keep his/her intention in the future [17]. However, committed voters who become disillusioned are assumed to retain their general party preference.
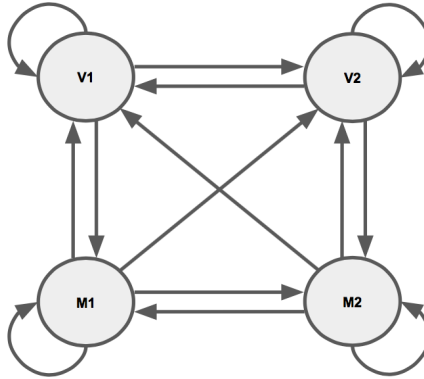
**Figure 1:** Graphical representation of movement within the system due to either media or interaction mechanisms.

We set our model as a discrete-time Markov chain, $\vec{\pi}(t+1) = \vec{\pi}(t)\mathbf{T}$, with $\vec{\pi} = (V_1, M_1, M_2, V_2)$ being the state vector. The transition matrix $\mathbf{T}$ is defined as follows:

$$\mathbf{T} = \begin{pmatrix} P_{V_1}^{V_1} & P_{M_1}^{V_1} & 0 & P_{V_2}^{V_1} \\ P_{V_1}^{M_1} & P_{M_1}^{M_1} & P_{M_2}^{M_1} & P_{V_2}^{M_1} \\ P_{V_1}^{M_2} & P_{M_1}^{M_2} & P_{M_2}^{M_2} & P_{V_2}^{M_2} \\ P_{V_1}^{V_2} & 0 & P_{M_2}^{V_2} & P_{V_2}^{V_2} \end{pmatrix}, \tag{1}$$

where $P_x^y$ is the probability of an individual from the superscript group, $y$, moving to the subscript group, $x$.

We define the outcome of the election as

$$V = V_1(t_E) - V_2(t_E),$$

where $t_E$ is the week of the election. This quantity is negative if the Democrat candidate wins, and positive if the Republican candidate does. The magnitude represents the difference between the percentage of votes each candidate got.

### 2.1.1  Person–to–person interaction model

Graphical representations of the possible movements within the system and the interactions under consideration are in Figures 1 and 2, respectively. We assume the following:

- All movements are influenced by person-to-person contact.

- All individuals are influential but to different degrees. In particular, people in the $V$ groups exert influence with more extreme effect than people in the $M$ groups. This does not assume that $V$ groups are more capable of changing the political affiliation of someone else. Instead, it indicates the possible extreme results that arise from interacting with $V$, e.g. $V_1 \rightarrow V_2$.

- The population mixes homogeneously, i.e., each individual is equally likely to make contact with any other individual. This means the chance of coming into contact with an individual in the $V_1$ population is $V_1/N$. This assumption is partially supported by the rise of social networks, in particular Facebook, which facilitate connection between people with all kinds of political opinions. The assumption of homogeneous mixing serves as a first step to facilitate analysis as done in [40, 51]. We aim to study the implications in situations that homogeneous mixing is a reasonable assumption. However, a more accurate description would likely include preferential mixing, in which a person is exposed more to similar opinions.

- It is possible for non-voting members to be influenced to vote and for voting members to stop voting.
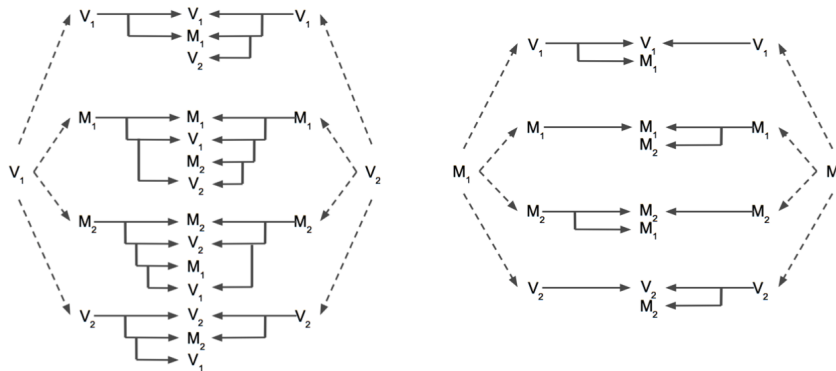


**Figure 2:** Person-to-person interaction and resulting effects: The dashed arrows represent the interaction and the solid lines represent the result(s) of the interaction.

To illustrate an interaction (which exists in Figure 2), take

$$V_1 \dashrightarrow M_1 \longrightarrow V_2.$$

This means "$V_1$ interacts with $M_1$ causing $M_1$ to change to $V_2$". We denote the probability of this transition as $f(V_2, M_1, V_1)$, i.e., the probability that an individual is in $V_2(t+1)$ given previously being in $M_1(t)$ and having an interaction with $V_1$ at time $t$. We can now update the definition of the probability of moving from group $y$ to group $x$ to account for all interactions:

$$P_x^y(t) = \sum_{k \in \vec{\pi}} \left( f(y(t+1), x(t), k(t)) \frac{k(t)}{N} \right),$$

where $\vec{\pi} = (V_1, M_1, M_2, V_2)$. Again, in general we leave $t$ dependence implicit.

For convenience, the following notation is defined for the parameters in the context of one time step:

- $\alpha_x^y$ : probability of an individual from the superscript group $y$ not moving after an interaction with an individual from the subscript group $x$—a failure to influence, where $x$ is the source of influence.

- $\beta_x^y$ : probability of an individual from the superscript group $y$ moving to a group of the same party, given that the interaction with an individual from the subscript group $x$ is a success, from $x$'s perspective in a time step.

- $\gamma_x^y$ : probability of an individual from group $y$ moving to the $M$-group of the other party, given that the interaction with group $x$ is a success.

All probabilities $P_x^y$ of the interaction transition matrix are as follows:

$$P_{V_i}^{V_i} = \alpha_{V_i}^{V_i} \frac{V_i}{N} + \alpha_{V_j}^{V_i} \frac{V_j}{N} + \alpha_{M_i}^{V_i} \frac{M_i}{N} + \frac{M_j}{N},$$

$$P_{V_i}^{M_i} = \left(1 - \alpha_{V_i}^{M_i}\right) \beta_{V_i}^{M_i} \frac{V_i}{N} + \left(1 - \alpha_{V_j}^{M_i}\right) \beta_{V_j}^{M_i} \frac{V_j}{N},$$

$$P_{V_i}^{M_j} = \left(1 - \alpha_{V_i}^{M_j}\right) \left(1 - \beta_{V_i}^{M_j}\right) \left(1 - \gamma_{V_i}^{M_j}\right) \frac{V_i}{N} + \left(1 - \alpha_{V_j}^{M_j}\right) \left(1 - \beta_{V_j}^{M_j}\right) \frac{V_j}{N},$$

$$P_{V_i}^{V_j} = \left(1 - \alpha_{V_i}^{V_j}\right) \left(1 - \beta_{V_i}^{V_j}\right) \frac{V_i}{N},$$

$$P_{M_i}^{V_i} = \left(1 - \alpha_{V_i}^{V_i}\right) \frac{V_i}{N} + \left(1 - \alpha_{V_j}^{V_i}\right) \beta_{V_j}^{V_i} \frac{V_j}{N} + \left(1 - \alpha_{M_i}^{V_i}\right) \frac{M_i}{N},$$

$$P_{M_i}^{M_i} = \alpha_{V_i}^{M_i} \frac{V_i}{N} + \alpha_{V_j}^{M_i} \frac{V_j}{N} + \frac{M_i}{N} + \alpha_{V_i}^{M_i} \frac{M_j}{N},$$

$$P_{M_i}^{M_j} = \left(1 - \alpha_{V_i}^{M_2}\right) \left(1 - \beta_{V_i}^{M_j}\right) \gamma_{V_i}^{M_j} \frac{V_i}{N} + \left(1 - \alpha_{V_j}^{M_j}\right) \frac{M_i}{N}.$$

### 2.1.2 Media model

This model assumes that members and voters switch class motivated only by the amount of favorable or unfavorable media coverage of either candidate. TV mentions of different candidates by source are to be found in [77] and reports from Pew Research Center [65] have quantified the amount of favorable or negative coverage of candidates over time. Based on these data, we define the following:

- $f_i(t)$ is the favorable coverage (e.g. TV mentions) for candidate $i$;

- $n_i(t)$ is the negative coverage for candidate $i$;

- $T(t) = f_1 + n_1 + f_2 + n_2$ is the total considered coverage at time $t$;

and

$$F_i(t) \equiv \frac{f_i(t)}{T(t)}, \quad N_i(t) \equiv \frac{n_i(t)}{T(t)}.$$

We assume that enough negative coverage of one candidate alone can make voters decay—loss of preference—to members ($V_i \to M_i$) or switch to vote for the opponent. Members can be recruited by favorable coverage of the candidate or by negative coverage of the opponent.

The parameters are defined as follow:

- $\sigma_i$ : effectiveness of $N_i$ on the movement from $V_i$ to $V_j$.

- $\delta_i$ : effectiveness of $N_i$ on the movement from $V_i$ to $M_i$.

- $\gamma_i$ : effectiveness of $N_j + F_i$ on the movement from $M_i$ to $V_i$.

- $\lambda_i$ : effectiveness of $N_i + F_j$ on the movement from $M_i$ to $V_j$.

- $\kappa_i$ : effectiveness of $N_i + F_j$ on the movement from $M_i$ to $M_j$.

The equations for passing from one class to the other are presented in Figure 3. Here $\delta_i$, $\gamma_i$, $\lambda_i$, $\sigma_i$, $\kappa_i$ are constants which play the role of amplification or reduction factors. This means that, for example, the amount of negative coverage about candidate $i$ drives a voter in $V_i$ towards $M_i$, but the effectiveness of the coverage depends on $\delta_i$. If $\delta_i = 0$, then individuals won't go from $V_i$ to $M_i$, no matter how much negative coverage the candidate $i$ has. Otherwise, the transition probability with respect to the amount of coverage will be linear. In a sense, they represent how "sensitive" a group is to the influence of the media.
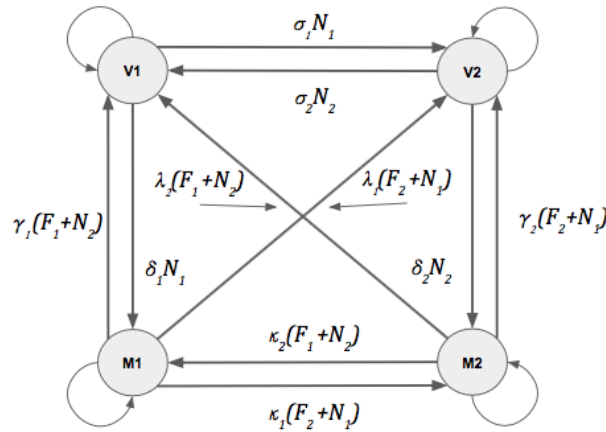
**Figure 3:** Media model—a map of movements within the system.

In this model, the transition probabilities depend on media coverage over time as follows:

$$
\begin{aligned}
P_{V_1}^{M_1} &= \gamma_1(F_1 + N_2), & P_{V_2}^{M_2} &= \gamma_2(F_2 + N_1), \\
P_{V_2}^{M_1} &= \lambda_2(F_2 + N_1), & P_{V_1}^{M_2} &= \lambda_2(F_1 + N_2), \\
P_{M_2}^{M_1} &= \kappa_1(F_2 + N_1), & P_{M_1}^{M_2} &= \kappa_2(F_1 + N_2), \\
P_{M_1}^{V_1} &= \delta_1 N_1, & P_{M_2,}^{V_2} &= \delta_2 N_2, \\
P_{V_1}^{V_2} &= \sigma_2 N_2, & P_{V_2}^{V_1} &= \sigma_1 N_1, \\
P_{V_1}^{V_1} &= 1 - \sigma_1 N_1 - \delta_1 N_1, & P_{V_2}^{V_2} &= 1 - \sigma_2 N_2 - \delta_2 N_2, \\
P_{M_1}^{M_1} &= 1 - \gamma_1(F_1 + N_2) - \kappa_1(F_2 + N_1) - \lambda_2(F_2 + N_1), \\
P_{M_2}^{M_2} &= 1 - \gamma_2(F_2 + N_1) - \kappa_2(F_1 + N_2) - \lambda_2(F_1 + N_2).
\end{aligned}
$$

Now let the parameter vector be defined as follows:

$$
\vec{p} \equiv \begin{bmatrix} \sigma_1 & \sigma_2 & \delta_1 & \delta_2 & \gamma_1 & \gamma_2 & \lambda_1 & \lambda_2 & \kappa_1 & \kappa_2 \end{bmatrix}.
$$

Then two constraints must be satisfied: first, $\vec{0} \leq \vec{p} \leq \vec{1}$; second, each row in the transition matrix sums to 1. This restriction can be stated by defining the matrix

$$
\mathbf{A} = \begin{bmatrix}
N_1 & 0 & N_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & N_2 & 0 & N_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & N_2+F_1 & 0 & N_1+F_2 & 0 & N_1+F_2 & 0 \\
0 & 0 & 0 & 0 & 0 & N_1+F_2 & 0 & N_2+F_1 & 0 & N_2+F_1
\end{bmatrix},
$$

and setting $\vec{0} \leq \mathbf{A}\vec{p}^T \leq \vec{1}$ in the parameter estimation process, so that the sum of the non-diagonal elements in each row of the matrix $\mathbf{T}$ is between 0 and 1.

### 2.1.3   Data collection

We use two different data sets (polling data) that contain information on voting preferences from the people who self-identify as either Democrat or Republican during the 2012 and 2016 presidential elections for parameter fitting. We further cross reference the credibility of each data source with the rating published by Nate Silver [76]. All polls have a rating of $A-$ or higher, with the exception of Politico/George Washington University with a $B$ rating.

Specifically, the first data set is compiled of weekly[1] data sets from multiple sources spanning 14 weeks from the beginning of August 2012 to right before the election day, November 6, 2012, from [52, 54, 57, 59, 66]. All polls are nationwide polls with the exception of one statewide poll, which still fits the trend of the data. The second set is collected through a single source. In this case, the data are taken weekly[2] between May 18 and July 12, 2016 from [69]. Details of the data sets used for the interaction model can be found in our supplementary material.

We also collect data on the number of times a candidate is mentioned on TV and headlines either favorably or unfavorably. For the 2012 election, data is taken from the studies in [65]. For 2016, the period from May to June is covered by the report in [64]. Data on the month of July is taken from [77] (analysis was performed in July 2016, at the end of the primary season).

We use voting data partially because it is one of the few and most easily accessible sources of data that focus on the voting intention of voters prior to the election. Even though polling data may contain significant errors and biases, it still has value since it should be in some proximity of the actual measurements. Moreover, our analysis deals only with the approximate number of people with intention to vote for one candidate or the other. Even if we had absolute certainty of the distribution of the voting intention on the day of the election, this would not guarantee that the results of the elections would match this distribution. The error between the actual Election Day result and the last poll data before that has remained consistent through the years [22], but we focus on the changes in voting behavior that most affect the proportion of voting intention.

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[1]We use Monday as a reference. If the data set is taken within that week, then we categorize it as data for that week.

[2]Wednesday is used as the reference similarly.

### 2.1.4    Model limitations

In relation to our data collection, the models are disproportionately complex, thus creating various problems in model justification/falsification, such as parameter identification. Thus, whatever conclusions made from studying our models and our data, further work needs to be done to examine them.

## 2.2    Computational methods

In order to determine the driving mechanisms behind an election, we first need to identify which parameter has more impact under each assumption. To do this, we use two different methods of parameter fitting and compare the results.

### 2.2.1    Parameter estimation

For the media model, we assume constant media coverage between consecutive data points. We then update the parameters using the favorable and unfavorable coverage data. The model is fitted through a minimization of the least squared error using a built-in MATLAB function ("fmincon") that allows for constraints on the minimizing parameters [29] to find the proportional constants that scale the effect of the media to changes in voting behavior. Note that a global minimum is not guaranteed. The objective that we aim to minimize in the estimation is the mean squared error in the proportional sizes $\vec{\pi}/N$ of each class, given in equation 2, where $s$ is the number of class or data set (one for each class), $\phi$ is the number of data points per time series data type,

$$E = \frac{1}{s\phi N} \sum_{j=1}^{s} \sum_{n=1}^{\phi} \left( \vec{\pi}_j(t_n) - \vec{\pi}_j^{\text{obs}}(t_n) \right)^2 . \tag{2}$$

For the person-to-person interaction model, we first apply the same scheme as above. However, due to the complexity of the transition matrix (state-dependency and non-linearity), many local error minima may occur. This could potentially give bad parameter estimates. Thus, we run the function multiple times with random initial guesses to obtain multiple sets of best fit parameters (with the same fitting error) for each data set and choose one that is representative of each set. Each set of parameters varies only slightly from one another. This suggests that we either obtain a global minimum or get stuck in a local minimum. Some of the variations in each set of parameters are artificial in the sense that they do not affect the fitting. More precisely, since the conditional probabilities $\beta$ and $\gamma$ only matter if the corresponding $\alpha$ is not 1, it means that some variations occur in $\beta$ and $\gamma$, when the corresponding $\alpha$ is 1.

### 2.2.2 Sensitivity analysis

To address the question of which transitions have the greatest impact on the election outcome, we carry out sensitivity analysis for both systems [53]. The outcome of the election is $V = V_1(t_E) - V_2(t_E)$, where $t_E$ is the week of the election (which we take to be the last time-step of each simulation). For the peer interaction model, we calculate the closed forms of the sensitivity of $V \equiv V_1(t+1) - V_2(t+1)$ with respect to each parameter between two consecutive time points. Recall that $V_1, V_2, M_1, M_2$ are taken to be evaluated at time $t$. The explicit equation for each sensitivity index can be found in the supplementary material.

To find the sensitivity values in the media model, we vary each parameter by 1% while fixing all of the others and use the model to recalculate V. Let the percent change in $V$ be

$$\%V = \frac{V_{est} - V_{rec}}{V_{est}},$$

where $V_{est}$ is the value of $V$ with the original estimated set of parameters and $V_{rec}$ is the value obtained after modifying the parameter. The sensitivity value is the ratio between percent change in $V$ and percent change in the parameter [6].

## 3 Results

### 3.1 Interaction models

#### 3.1.1 2012 election

For the interaction model, the best fit curves of the 2012 poll data are shown in Figure 4 with SSE (sum of squared errors) of 0.0149. The parameter estimates are shown in Table 1. Interestingly, there are many zero and one probabilities. These parameter values define a set of implications on the result of interactions between two individuals in the system. These are explained in full below and collectively visualized in Figure 11. Note that for the following, the usage of *always* and *never* are approximations by our model. The following conclusions only address the result of one encounter between two individuals in a single time-step.

Since $\alpha_{V_1}^{V_1}, \alpha_{V_2}^{V_1} > 0.5$, $\alpha_{M_1}^{V_1} = 0$, and $\alpha_{M_2}^{V_1} = 1$, a Romney voter (a person who declares to be voting for Romney, regardless of party affiliation): *usually* remains a Romney supporter following an interaction with either another Romney voter or Obama voter; *never* remains a supporter of Romney following an interaction with a non-voting Republican member (that is, a person who

self-identifies with the Republican party but does not wish to support Romney); *always* remains a supporter of Romney following an interaction with a Democratic member.

Since $\alpha_{V_2}^{V_2} = 1$, $\alpha_{V_1}^{V_2} > 0.5$, $\alpha_{M_1}^{V_2} = 0$, and $\alpha_{M_2}^{V_2} < 0.5$, an Obama voter: *always* remains a supporter of Obama following an interaction with another Obama voter; *usually* remains a supporter of Obama following an interaction with a Romney voter; *usually doesn't* remain a supporter of Obama following an interaction with a Democratic member; *never* remains a supporter of Obama following an interaction with a Republican member.

Since $\alpha_{V_1}^{M_1} < 0.5$ and $\alpha_{V_2}^{M_1} = 0$, a Republican member: *usually doesn't* stay uncommitted after an interaction with a Romney voter; *never* stays uncommitted following an interaction with a Obama voter.

Lastly, $\alpha_{V_1}^{M_2} = 0$ and $\alpha_{V_2}^{M_2} = 1$, so a [non-voting] Democratic member: *never* stays uncommitted following an interaction with a Romney voter; *always* stays uncommitted following an interaction with a Obama voter.
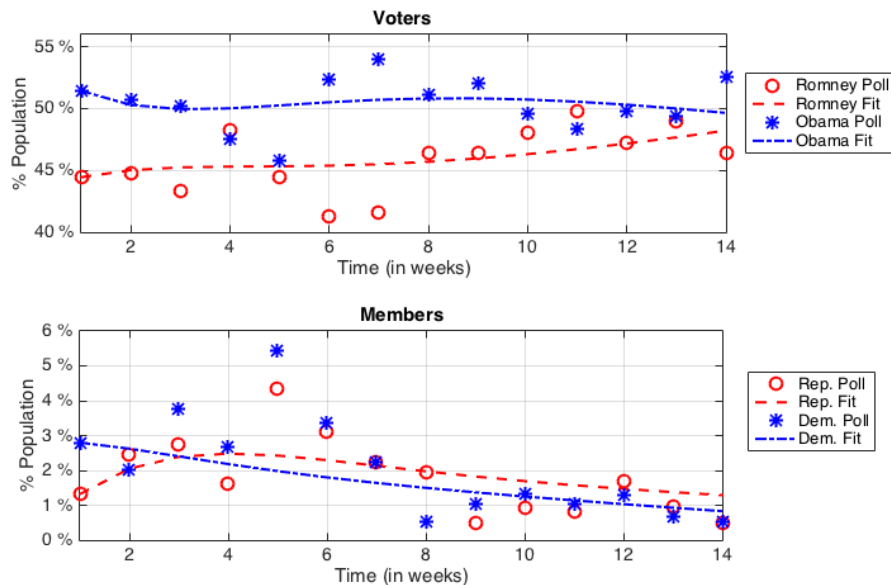


**Figure 4:** Interaction model and best fit curves for the 2012 election: The plot on the top shows the modeled voting classes, $V_1$ and $V_2$, and the bottom plot presents the modeled member classes, $M_1$ and $M_2$.

**Table 1:** Parameter estimates and sensitivities for the interaction model applied to the 2012 election.

| Parameter | Estimation | Sensitivity | Parameter | Estimation | Sensitivity |
|---|---|---|---|---|---|
| $\alpha_{V_1}^{V_1}$ | 0.9942 | -3.905 | $\alpha_{V_2}^{V_1}$ | 0.9398 | -7.432 |
| $\alpha_{V_1}^{M_1}$ | 0.02112 | -0.002 | $\beta_{V_2}^{M_1}$ | 0 | 0 |
| $\alpha_{V_1}^{M_2}$ | 0 | 0 | $\alpha_{V_2}^{M_2}$ | 1 | 0.044 |
| $\alpha_{V_1}^{V_2}$ | 0.8840 | 6.990 | $\alpha_{V_2}^{V_2}$ | 1 | 3.980 |
| $\alpha_{M_1}^{V_1}$ | 0 | 0 | $\alpha_{M_2}^{M_1}$ | 1 | 0 |
| $\alpha_{M_2}^{M_2}$ | 0 | 0 | $\alpha_{M_2}^{V_2}$ | 0.2307 | 0.013 |
| $\beta_{V_1}^{M_1}$ | 0 | 0 | $\beta_{V_2}^{V_1}$ | 0 | 0 |
| $\beta_{V_1}^{M_2}$ | 0 | 0 | $\beta_{V_2}^{M_1}$ | 0 | 0 |
| $\beta_{V_1}^{V_2}$ | 0 | 0 | $\beta_{V_2}^{M_2}$ | 0.1051 | 0 |
| $\gamma_{V_1}^{M_2}$ | 1 | 0.055 | $\gamma_{V_2}^{M_1}$ | 0 | 0 |

Substituting the estimated parameters in the transition matrix, we obtain

$$T = \begin{pmatrix} \frac{0.99V_1 + M_2 + 0.96V_2}{N} & \frac{0.01V_1 + M_1}{N} & 0 & \frac{0.04V_2}{N} \\ 0 & \frac{0.02V_1 + M_1 + M_2}{N} & 0 & \frac{0.98V_1 + V_2}{N} \\ 0 & \frac{V_1 + M_1}{N} & \frac{M_2 + V_2}{N} & 0 \\ \frac{0.09V_1}{N} & 0 & \frac{0.77M_2}{N} & \frac{0.91V_1 + M_1 + 0.23M_2 + V_2}{N} \end{pmatrix}.$$

This gives some important results about the fitted model: Republican members cannot directly (in one time step) become Democrat members; there is no direct recruitment of party members to party voters within either party; Democrat members can only change to become Republican members; Republican voters usually drives Republican members away to vote for the other party; when voters from different parties interact, the Democrat voter is around twice as likely to become a Republican voter than the Republican is to switch candidates; Democrat voters can decay to members by interaction with Democrat members, while Republican voters can decay by interaction with both Republican members and voters.

The sensitivity analysis, see Table 1 and Figure 5, reveals that $\alpha_{V_2}^{V_1}$ and $\alpha_{V_1}^{V_2}$, which represent the probability that a voter interacts with a voter of the opposite party and doesn't change class, have the most impact on the outcome of the election. Other than these, the parameters with higher sensitivity are $\alpha_{V_1}^{V_1}$ and $\alpha_{V_2}^{V_2}$ (i.e. probability of voters staying the same given that they interact within their own class). Note that the interactions that most affect the change in the
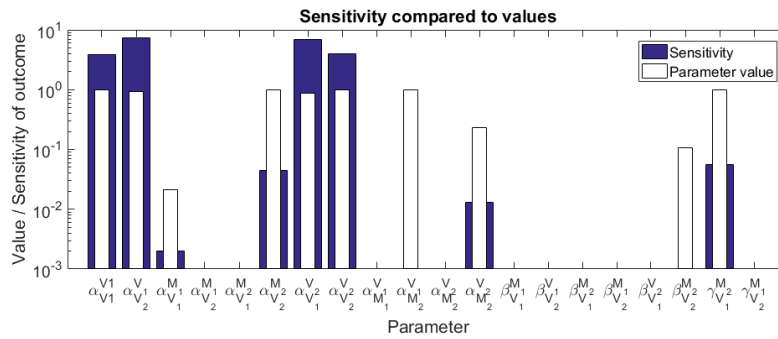
**Figure 5:** Interaction model and bar plot for the contrast of parameter sensitivity to the parameter value for the 2012 election: Y-axis is in log-scale.

final vote are meetings between voters of each party; the election is strongly dependent on how likely voters are to fail to change each other's minds.

### 3.1.2   2016 election

The best fit curve for the 2016 poll data is shown in Figure 6 with SSE of 0.0037. The parameter estimates and the sensitivity in the last time step are recorded in Table 2. Like the 2012 election parameter values, the 2016 election parameters values have meanings that are explained below and visualized in Figure 11.

Since $\alpha_{V_1}^{V_1} = 1, \alpha_{V_2}^{V_1} < 0.5, \alpha_{M_1}^{V_1} = 1$, and $\alpha_{M_2}^{V_1} = 1$, a voter for Trump: *always* remains a supporter of Trump due to interaction with either another Trump voter or Republican or Democratic member; *usually doesn't* remain a supporter of Trump after an interaction with a Clinton voter.

Since $\alpha_{V_2}^{V_2} = 1, \alpha_{V_1}^{V_2} < 0.5, \alpha_{M_1}^{V_2} = 0$, and $\alpha_{M_2}^{V_2} = 1$, a voter for Clinton: *always* remains a supporter of Clinton after an interaction with either another Clinton voter or non-voting Democratic member; *usually doesn't* remain a supporter of Clinton after an interaction with a Trump voter; *never* remains a supporter of Clinton after an interaction with a Republican member.

Since $\alpha_{V_1}^{M_1} = 0$ and $\alpha_{V_2}^{M_1} = 0$, a Republican member: *never* stays uncommitted after an interaction with a Trump voter; *never* stays uncommitted after an interaction with a Clinton voter.

Lastly, $\alpha_{V_1}^{M_2} = 1$ and $\alpha_{V_2}^{M_2} = 0$, so a Democratic member: *always* stays uncommitted after an interaction with a Trump voter; *never* stays uncommitted after an interaction with a Clinton voter.
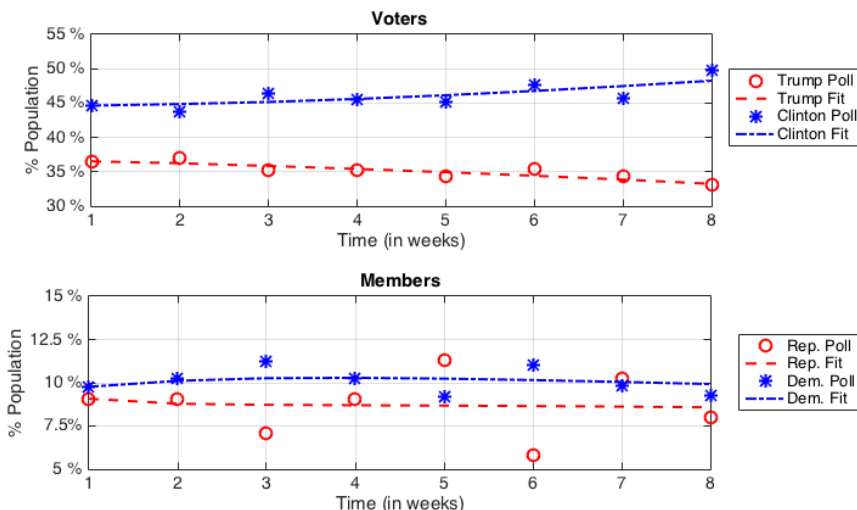
**Figure 6:** Interaction model and best fit curves for the 2016 election: The plot on the top shows the modeled voting classes, $V_1$ and $V_2$, and the bottom plot presents the modeled member classes, $M_1$ and $M_2$.

The transition matrix we derive with the parameter values is

$$
T = \begin{pmatrix}
\frac{V_1+M_1+M_2+0.47V_2}{N} & \frac{0.38V_2}{N} & 0 & \frac{0.15V_2}{N} \\
\frac{V_1}{N} & \frac{M_1+M_2}{N} & 0 & \frac{V_2}{N} \\
0 & \frac{M_1}{N} & \frac{V_1+M_2}{N} & \frac{V_2}{N} \\
\frac{0.32V_1}{N} & 0 & \frac{0.34V_1}{N} & \frac{0.34V_1+M_1+M_2+V_2}{N}
\end{pmatrix}.
$$

Again, we can read some information from the transition matrix, written in contrast to the 2012 election transition matrix (section 3.1): Republican members cannot directly (in one time step) become Democrat members; there is direct recruitment of party members to party voters within either party; if voters from different parties interact, the Democrat is still nearly twice as likely as the Republican to switch candidates; both voter groups only decay to their party's member groups by interaction with the other voter group; every transition between different groups is solely caused by a voter class, except for the case in which Republican members influence Democratic members to become Republican members.

Note also that Republican voters no longer influence voters or members in their own party to stop supporting the candidate or leave the party. Yet, they also no longer influence Democratic members to become Republican members.

**Table 2:** Parameter estimates and sensitivities for the interaction model applied to the 2016 election.

| Parameter | Estimation | Sensitivity | Parameter | Estimation | Sensitivity |
|---|---|---|---|---|---|
| $\alpha_{V_1}^{V_1}$ | 1 | 0.235 | $\alpha_{V_2}^{V_1}$ | 0.4670 | 0.188 |
| $\alpha_{V_1}^{M_1}$ | 0 | 0 | $\beta_{V_2}^{M_1}$ | 0 | 0 |
| $\alpha_{V_1}^{M_2}$ | 1 | 0.013 | $\alpha_{V_2}^{M_2}$ | 0 | 0 |
| $\alpha_{V_1}^{V_2}$ | 0.3441 | -0.159 | $\alpha_{V_2}^{V_2}$ | 1 | -0.417 |
| $\alpha_{M_1}^{V_1}$ | 1 | 0.070 | $\alpha_{M_2}^{M_1}$ | 1 | 0 |
| $\alpha_{M_2}^{M_2}$ | 0 | 0 | $\alpha_{M_2}^{V_2}$ | 1 | -0.093 |
| $\beta_{V_1}^{M_1}$ | 1 | 0.140 | $\beta_{V_2}^{V_1}$ | 0.7123 | 0.119 |
| $\beta_{V_1}^{M_2}$ | 0.4625 | 0 | $\beta_{V_2}^{M_1}$ | 0 | 0 |
| $\beta_{V_1}^{V_2}$ | 0.5228 | -0.107 | $\beta_{V_2}^{M_2}$ | 1 | -0.187 |
| $\gamma_{V_1}^{M_2}$ | 0.4852 | 0 | $\gamma_{V_2}^{M_1}$ | 0 | 0 |

The sensitivity analysis, see Table 2 and Figure 7, reveals the parameters to which the outcome of the final poll is most sensitive: $\alpha_{V_2}^{V_2}$ (the probability that $V_2$ stays $V_2$ given an interaction with $V_2$), $\alpha_{V_1}^{V_1}$, $\alpha_{V_2}^{V_1}$, and $\beta_{V_2}^{M_2}$. As in the 2012 election interaction model, we find that the most influential parameters are those associated with the likelihood of voters failing to change other voters' minds.
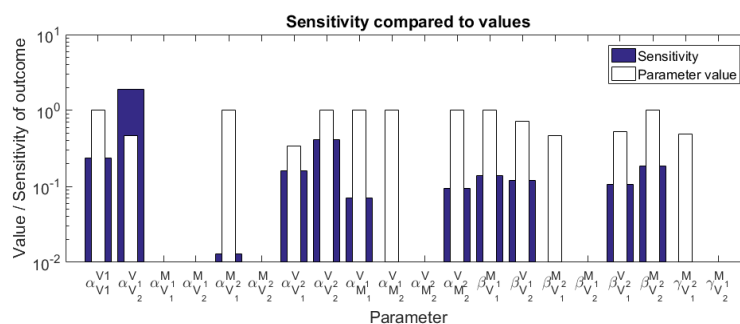


**Figure 7:** Interaction model and bar plot for the contrast of parameter sensitivity to the parameter value for the 2016 election: Y-axis is in log-scale.

## 3.2   Media models

### 3.2.1   2012 election

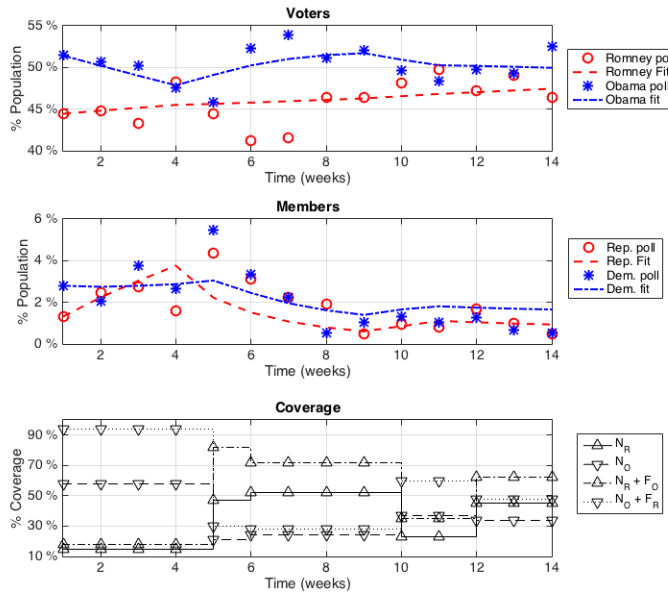Parameter estimation for the media model produces the best-fit line in Figure 8.



**Figure 8:** Media model and best fit curves for the 2012 election: The plot on the top shows the modeled voting classes, $V_1$ and $V_2$; the middle plot presents the modeled member classes, $M_1$ and $M_2$; the bottom plot displays the amount of media coverage at a given time. For each candidate, we present the percentage of negative coverage affecting him and the added percentage of his own negative coverage plus the positive coverage for the competitor.

Table 3 presents the results. The parameter with the highest fitted value is $\gamma_2$, which means that the media effect has more influence in the Democrat members' recruitment (i.e. from $M_2$ to $V_2$) than in other movements. The parameters with lower values reveal the transitions in which the media effect has a lower impact. In this case, there are four parameters with values of order $10^{-5}$ or $10^{-6}$, namely $\gamma_1, \lambda_2, \delta_1$ and $\sigma_1$. The corresponding transitions with low media susceptibility are: recruitment ($M_1$ to $V_1$) inside the Republican party; recruitment ($M_2$ to $V_1$) outside the Republican party; decaying ($V_1$ to $M_1$) inside the Republican party; switching ($V_1$ to $V_2$) from voting Republican to voting Democrat.

**Table 3:** Parameter estimates and sensitivities for the media model applied to the 2012 election. The SSE for the estimation is 0.0137.

| Parameter | Estimate | Sensitivity | Parameter | Estimate | Sensitivity |
|-----------|----------|-------------|-----------|----------|-------------|
| $\delta_1$ | 8.90e-6 | -0.0011 | $\delta_2$ | 4.68e-2 | 0.8290 |
| $\gamma_1$ | 5.35e-5 | 0.0004 | $\gamma_2$ | 8.47e-1 | -0.9187 |
| $\lambda_1$ | 2.35e-4 | -0.0003 | $\lambda_2$ | 6.13e-5 | 0.0006 |
| $\sigma_1$ | 9.69e-6 | -0.0016 | $\sigma_2$ | 1.35e-2 | 2.2896 |
| $\kappa_1$ | 6.85e-1 | -0.4012 | $\kappa_2$ | 4.25e-1 | 0.3321 |

We wish to know which parameters have more impact in determining the final result of the election. We vary each estimated parameter individually by a few percent and calculate the effect it has on the outcome of the election, namely $V_1 - V_2$. The calculated sensitivity indices are presented in Table 3 and Figure 10. Parameters driving transitions which decrease the number of Republican voters and/or increase the number of Democratic voters naturally have negative sensitivity indices for $V$.

First note that the five parameters with lower values are actually the ones to which the outcome is less sensitive. However, the largest parameter, $\gamma_2$, does not produce the highest sensitivity value: $\sigma_2$, which is not among the highest values, does. This last fact suggests that the outcome of the election was most affected by how likely negative coverage for the Democratic candidate is to drive Democrat voters to vote Republican. The parameter $\gamma_2$ being the largest means that recruitment in the Democrat party is the transition most affected by media.

The next values in magnitude are the $\kappa$ parameters, which determine movement between the member classes. It follows that, in a scenario with equal amount of negative and positive media for each candidate, the most likely transitions will be a member switching and Democrat recruitment. Republican voters, on the other hand, are not likely to change parties or decay due to media influence (since $\sigma_1$ and $\delta_1$ are below $10^{-5}$).

When compared to the equivalent processes of decay and switching in the Democratic party, the probabilities could as well be neglected (smaller by about four orders of magnitude). Notice that these precise transitions in the Democrat party have the most impact on the outcome: $\sigma_2$ and $\delta_2$ have the larger sensitivity values, which means that the most effective way to change the result is by making Democrat voters more likely to respond to coverage against Obama and change class. Of course, this kind of change is difficult to actually achieve, but it can be addressed by changing the media content to target specifically Democrat voters and cause a bad image of Obama based on issues that Democrats care about.

The general qualitative behavior of the system can be observed in the left diagram in Figure 12. A first conclusion is that a given amount of coverage causes more movement inside the Democratic party than inside the Republican one. It also appears that Democrat voters are more likely than Republicans to switch candidates because of negative media coverage.

### 3.2.2   2016 election

**Table 4:** Parameter estimates and sensitivities for the media model applied to the 2016 election. The SSE for the estimation is 0.0039.

| Parameter | Estimate | Sensitivity | Parameter | Estimate | Sensitivity |
|-----------|----------|-------------|-----------|----------|-------------|
| $\delta_1$ | 3.66e-2 | -0.2171 | $\delta_2$ | 7.95e-4 | 0.0038 |
| $\gamma_1$ | 4.2e-3 | 0.0081 | $\gamma_2$ | 8.02e-2 | -0.2023 |
| $\lambda_1$ | 2.3e-3 | -0.0053 | $\lambda_2$ | 3.5e-3 | 0.0081 |
| $\sigma_1$ | 8.81e-4 | -0.0102 | $\sigma_2$ | 5.98e-4 | 0.0061 |
| $\kappa_1$ | 8.48e-1 | -0.0814 | $\kappa_2$ | 8.06e-1 | 0.0717 |

The weekly poll data for the 2016 election (from the third week of May to the second week of July) are best fit by the lines in Figure 9 with the parameter values given in Table 4. Notice that media favoring Clinton is consistently greater than media favoring Trump, which is reflected in the increasing behavior of the Democrat voters' fit. Correspondingly, the fit for Trump voters appears to be monotonically decreasing.

The parameters of higher value for this data set are the $\kappa$'s, which reveals a strong effect of media coverage in transitions between party members. The value of $\kappa_1$ being slightly greater indicates that, given the same amount of positive and negative coverage for both candidates, Republican members are more likely to become members of the other party than Democrat members.

The transitions in which media have less effect are the direct movements between the voting classes. Keep in mind that the model assumes that this transition occurs solely because of the amount of negative coverage of a candidate and is not dependent on the amount of positive coverage the other candidate is receiving (see Figure 3). In other words, negative coverage about the candidates is not very likely to drive voters to vote for the other candidate. In the absence of positive coverage, for example, a given equal amount of negative coverage for both candidates is roughly a thousand times more likely to cause movement between member classes than between voting classes. Notice we do not mean *net movement*, which may be zero by cancellation.
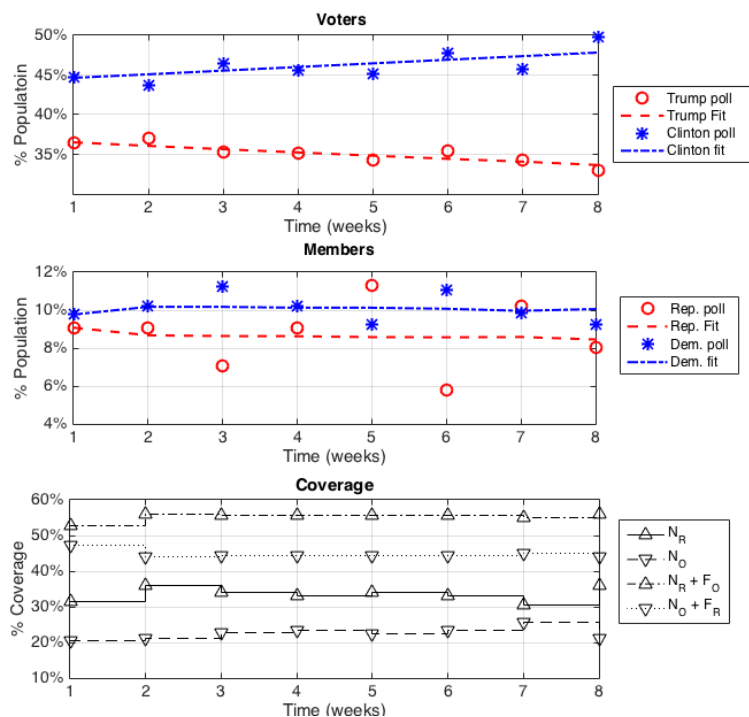
**Figure 9:** Media model and best fit curves for the 2016 election: The plot on the top shows the modeled voting classes, $V_1$ and $V_2$; the middle plot presents the modeled member classes, $M_1$ and $M_2$; the bottom plot displays the amount of media coverage at a given time. For each candidate, we present the percentage of negative coverage affecting him and the added percentage of his own negative coverage plus the positive coverage for the competitor.

Inside the parties, the media seem to have asymmetric effects. In the Republican party, media coverage affects the decaying (from Republican voter to member) more than the recruitment (from Republican member to voter). Since media influence is the only factor affecting the transitions in the model, at any given time we observe a probability of decay (choosing not to vote) of approximately 10 times greater than the recruitment probability. In the Democrat party, however, the probability of recruitment is more than 10 times greater than the probability of decaying.
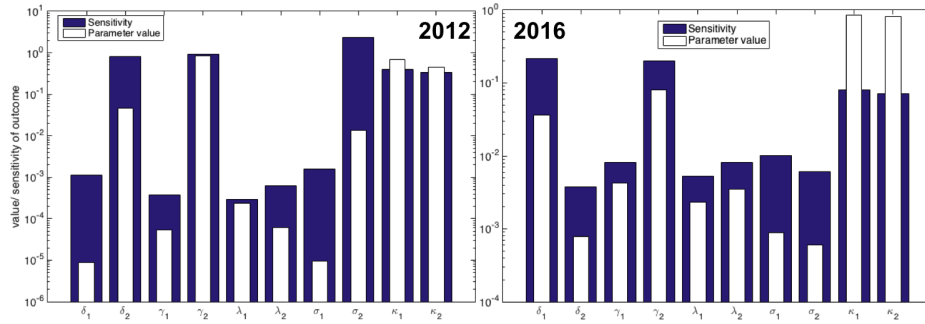
**Figure 10:** Media model and bar plot for the contrast of parameter sensitivity to the parameter value for the 2012 (left) and 2016 (right) election. Y-axis is in log-scale.
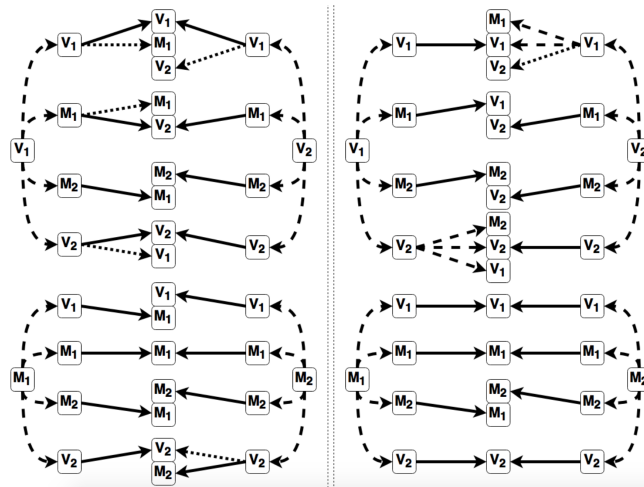


**Figure 11:** Person-to-person interaction maps comparison: 2012 (left) and 2016 (right) accounting for fitted parameters. Curved dashed lines represent interactions. Dotted lines, dashed lines and solid lines represent the transition with probability less than 0.3, 0.5 and 1, respectively. If a solid line represents the only possible result, then that transition has probability of 1.

# 4  Conclusions

In the context of person-to-person interaction, the level of influence by a group is qualitatively defined to be its constituents' ability to cause a *change-of-group* in an individual upon contact (regardless of the direction of influence).

Using Figure 11 as a reference, we observe that the voter groups are the driving forces behind the 2016 election. On the other hand, the 2012 election is characterized by a share in driving power between all groups with the voter groups having a slightly bigger effect on the system than the non-voting groups.

In particular, during the 2012 election, the influence of the Democratic voters, $V_2$, is weaker in comparison to the Republican voters, $V_1$. This characteristic is coupled with the observation that the voter groups are more likely to be persuaded to switch groups by the non-voting members of the party they support by one another. Interestingly, this changes completely in the 2016 election, when these voting groups are impartial to the influence of the non-voting groups, yet they are significantly influenced by one another. We propose the following interpretation of what these characteristics mean in term of the 2012 and 2016 elections.

For the 2012 presidential election, the interaction model shows that an interaction between a voting group and a non-voting group generally results in a loss of constituents for the voting group. This suggests that the voter turnout in the 2012 election should be less than the usual trends, regardless of the outcome of the election. This is consistent with what was observed in 2012. The voter turn-out rate with respect to the total eligible voters in 2012 was 57.5%, lower than in 2008 (62.3%) and 2004 (60.4%) [25].

In contrast, we observe different dynamics behind the 2016 election. The election is driven by the influence of the voting groups. Specifically, the Republican voters have a significant influence on the Democrat voters, while they have no influence over the non-voting Democrat members. Democrat voters don't have influence on other Democrat voters (they don't repel them) and they do drive members of both parties to vote Democrat. This can be seen from the number of possible outcomes with significant probability (greater than $0.3$) of happening after a Democratic voter meets a Republican voter, and vice versa. This number is two for the 2012 election and five for the 2016 election.

Note that the Democratic voters have a slightly weaker influence over the Republican candidate supporters compared to the influence that the Republican voters have over the Democratic voters. Yet, in 2012, Democratic voters exhibit absolute influence over Republican members to become Democratic voters, and, in 2016, they have this influence on both member groups.

On the other hand, the influence of the non-voting groups in the 2016 election is weaker compared to their influence in the 2012 election. In fact, in the 2012 election $M_1$ and $M_2$ can influence $M_1$, $M_2$, and even $V_1$, and $V_2$ to change their group affiliation. But in this election, their influence is limited to only the non-voting groups.
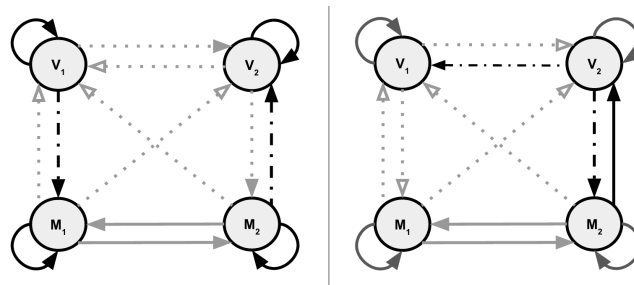
**Figure 12:** Resultant media flow diagrams for 2012 (left) and 2016 (right) elections. Line style represent fitted parameter values: solid $(> 10^{-1})$, dotted $(< 10^{-4})$, and dotted-dash $(10^{-4}, 10^{-1})$. Arrow color represents sensitivity values: black $(> 0.5)$, gray $[10^{-2}, 0.5]$ and white $(\leq 10^{-2})$.

As for the media effect on voting behaviors, there are some subtle but important changes between the two elections. First, note that overall, the media influence is noticeably more effective among the Republican members and voters in the 2016 presidential election than it was in the 2012 presidential election. This observation comes directly from comparing the magnitude of the parameters with index of 1 (movements from the Republican members and voters) between the two elections. Among the Democratic members and voters, the overall media influence is slightly weakened in the 2016 election compared to the 2012 election, e.g. three parameters decrease and two increase. Yet, the increase in sensitivity toward the media among the Republicans is minimal. The only parameter that increases significantly is the probability that voters stop supporting the Republican candidate $(\delta_1)$. Note that this is not equivalent to stating that the 2016 Republican candidate is losing because we only consider members of the two parties, and there is a significant percentage of voters who do not belong to either party [41].

Unlike the person-to-person interaction model where about half of the parameters have a significant value (larger than 0.05), only three of the parameters for the media model are significant $(\kappa_1, \kappa_2$ and $\gamma_2)$. The significance of these values means that the nonvoting members of both parties are more susceptible to the media influence. The difference is that the Republican members do not generally decide to vote because of media influence. They simply switch sides to identify with the other party, whereas the Democratic members either exhibit a similar behavior or simply decide to vote for the Democratic candidate. The high value of $\kappa$ is likely because the media tends to not actively encourage people to vote. Instead, it simply spreads news. On the other hand, the significance in value of $\gamma_2$ is probably caused by something else, since it is not consistent in both elections. With respect to some of the smaller parameters, we notice that

in 2012, there is a considerable tendency ($\sigma_2$ and $\delta_2$) for the Democrat voters to switch sides to vote for the Republican party or become non-voting members in response to negative news about the Democratic candidate. This observation is partially supported by the event when voters for Obama plummeted after he was perceived to have lost the first debate to Romney on October 3, 2012 [4, 26].

Furthermore, the slight decrease in sensitivity of the Democratic voters and members in 2016 is evident. This is because regardless of the huge amount of news covering the case of Clinton's email-handling, over two thirds of Democrats did not seem to think of it as a major problem compared to a near unanimous disapproval among the Republicans, who thought Clinton should have been charged for her mishandling of the confidential emails [20, 50]. Understanding this effect is critical since the media are always an important source of information for voters—especially with the 2016 election marked by multiple controversial and complex issues such as the economy, immigration and terrorism [12, 13].

In conclusion, the media and interaction models reveal some general aspects of the 2012 election. Both models were able to fit the data well, where the fit using the media model gave a smaller error than the fit using the person-to-person model. Since the media model contains fewer parameters than the interaction model, it suggests that if we compare the two models using an information criterion (such as the Akaike information criterion or the Bayesian information criterion), then we may find that the media model is better at fitting than the interaction model. The difference, however, is insignificant and could have easily been caused by artificial effects of parameter fitting and random errors. Thus, we refrain from concluding which best describes the most important factor in each election. Instead, we suggest a more thorough analysis to be done with more consistent data sets and similar numbers of parameters in both models.

The key elements of both models are the following:

- Media is most responsible for the transition between members of the two parties in both elections. The influence of media seems to be insignificant in all other transitions with one exception in 2012, which is the value of $\gamma_2$.

- The person-to-person interaction model suggests that the voter groups drove the 2016 election. This holds for the 2012 election but to a lesser extent.

- The person-to-person interaction model suggests that members in both party generally did not like Donald Trump. This comes from the observation that it is easier to influence them to change their minds to vote against

Trump and the increase in the effectiveness of media influence among Republicans in 2016 (note that Trump has high amount of negative coverage).

Theoretically, the transition that is most sensitive to the election outcome is between voters of the two groups. Thus convincing voters to switch side should be an effective strategy to gain an advantage in the election. Yet, this is not entirely feasible realistically. The key notes suggest that the most effective way realistically to drive the 2016 election is to manage the unfavorable opinions of members of both parties about Trump in their favor.

There are two major sets of limitations in our work. The first corresponds to the assumptions of our models and the second is related to our method and the available data. The political voting system is extremely complex, and naturally the presented models are only rough approximations of reality in order to measure the fundamental driving forces behind a presidential election. Applied to the 2012 and 2016 elections, sometimes, our models gave insights into the underlying mechanisms behind each election, which is supported by data. Other times, it gave suggestions that seem inconsistent with reality.

The second limitation is our collection of data. Aside from the random error that is inherent to polling data, we also use data sets from multiple sources for fitting of the 2012 election. This could have a major impact on our analysis. Yet, in our case, the effect seems to be minimal judging from the fitting errors and the trends of the data points.

As mentioned previously, our use of the data mainly bases on the number of self-identified likely voters for each party. This is appropriate for our purpose; however, for a more thorough analysis of the underlying social and media effects, a more detailed data set could be used. For example, a data set with information on the ideology of the likely voters can help us better identify theoretically the different directions of influences.

Besides using more accurate data and methods to analyze individual elections, further work should be done to study how elections affect voting dynamics. Our framework could be expanded towards comparing voting behavior across multiple campaign elections, incorporating a *cascade effect* [16] in which campaigning efforts in a given year can be carried on to the next election and influence the voting population. One last note on our framework: is it really appropriate to model voting behaviors using a Markov chain? Both models assume what happens in the future depends only on what is going on in the present. This is arguable because a decision can be viewed as the output of a collection of information. Thus, we also hope to extend this framework to a non-memoryless process. Furthermore, a more reasonable comparison between the effect of media and person-to-person interaction would require the incorporation

of both effects into one model. Combining both effects together in one model would create a far more complex model, thus care should be taken to simplify the model; one possible technique involves compressing the parameter space by looking at the eigenvalues of the Fisher Information Matrix [27].

Finally, the analysis in this study leaves out both the specific incidents in the last month of the 2016 campaign (such as the recording of Trump's lewd conversation with Billy Bush, or James Comey's announcement extending the FBI investigation of Clinton's email scandal), as well as any explicit consideration of interference in the election, all of which clearly had significant effects on the election's outcome.

## Acknowledgements

# References

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