

DYNAMIC ECONOMIC GROWTH: A NEW METHOD OF APPROACH

CRECIMIENTO ECONÓMICO DINÁMICO: UN NUEVO MÉTODO DE ENFOQUE

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Abstract

This paper looks at Harrod's fundamental assumptions and equations in 'An essay in dynamic theory' as the foundation for suggesting a better understanding of his postulates on economic growth. The analysis we present attempts to measure the effects of resource composition on the field of economic growth by determining its *elasticity* and the *coefficient of acceleration*. By addressing new critical assumptions, key formulas, and methods of approach, some crucial findings provide a strong theoretical model with which to examine dynamic economic growth. Furthermore, this paper offers a brief review of Solow's and Swan's critiques of Harrod's essay. This analysis could inspire further advances in the study of economic growth at the academic level and even in terms of government macroeconomic policies and business decisions.

Keywords: Cobb and Douglas; Keynes; Harrod; Domar; Swan; Solow; Warranted/Natural rates of growth; Effective/Aggregated demands.

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Resumen

Esta investigación se basa en los supuestos y ecuaciones fundamentales de Harrod en 'Un ensayo sobre teoría dinámica' como núcleo central para sugerir una mejor comprensión de sus postulados sobre el crecimiento económico. El análisis que presentamos pretende medir los efectos de la composición de los recursos en el crecimiento económico determinando su *elasticidad* y el *coeficiente de aceleración*. Al abordar nuevos supuestos críticos, fórmulas clave y métodos de enfoque, hallazgos cruciales proporcionan un modelo teórico sólido para examinar el crecimiento económico dinámico. Además, esta investigación ofrece una breve revisión de las críticas de Solow y Swan a ese escrito de Harrod. Esta investigación podría inspirar mayores avances en el estudio del crecimiento económico al nivel académico e inclusive en términos de política macroeconómica de gobierno y decisiones empresariales.

Palabras clave: Cobb and Douglas; Keynes; Harrod; Domar; Swan; Solow; Tasas de crecimiento requerido/natural; Demanda efectiva/agregada.

Introduction

To Harrod (1939) a dynamic theory necessarily involves thinking dynamically; it requires a *mental revolution*. His *tentative and preliminary attempt* is to “provide a framework of concepts relevant to the study of change” (p. 14). The axiomatic basis of his theory proposes that the level of a community’s income is the most important determinant of its supply of saving, that its rate of increase determines its demand for saving, and that both are equal. Harrod’s *relevant concepts of warranted and natural* rates of growth were questioned by Solow (1956) who understood that Harrod’s warranted concept is strictly related to full employment of the total labor force at each instant, which is not at all the case. In Harrod dynamic system, *warranted* means an *unstable system, a system that oscillates, a moving equilibrium of advance*, for which certain conditions are needed.

In such a system, “the trend of growth may itself generate forces making for oscillation (...) maintaining a trend of increase” (Harrod, 1939, p. 16) at a *warranted* rate; This author suggested that the value of capital goods (both fixed and circulating) “depends on the state of technology and the nature



of the goods constituting the increment of output” but “not the whole of the new capital is destined to look after the increment of output of consumers’ goods” (Harrod, 1939, p. 17). The author begins his analysis by suggesting that producers are satisfied having produced *neither more nor less than the right amount maintaining the same rate of growth*. For it, “stock in hand and equipment available will be exactly at the level which they would wish to have them” (Harrod, 1939, p. 22). In this quoted text the word ‘available’ means ‘able to be used’, ‘disposable’.

By relaxing the *‘for the moment’* statement, it follows that there is no unique warranted rate; it depends upon the phase of the trade cycle and the level of activity and at full employment there exist a warranted rate ‘proper’ to the economy. Harrod’s dynamic theory is an attempt to answer Keynes’s (1964, p. 4) following concern: “The question [...] of the volume of the *available* resources [...] has often been treated descriptively. But the pure theory of what determines the *actual employment of the available resources* has seldom been examined in great detail”. In view of foregoing, the main objective of this article is to provide a better understanding of Harrod’s *dynamic theory* by developing a *new method of approach* based on *thinking in terms of trends of increase* to reveal the divergence and/or convergence processes of natural and/or warranted rates of growth.

Harrod (1960) describes a dynamic theory as a set of relations between rates of increase (or decrease) of certain magnitudes, which, in turn, are thought of as *laws expressing certain necessary relations*. Based on this, we construct new formulas denoting relations between production, capital, labor and saving to achieve a new method of approach. The following two sections present the *coefficient of acceleration* and the resource composition dynamic to explain the natural and/or warranted trends of production growth. The concept of *elasticity* or *velocity* as it is relates to resource composition completes our fundamental theoretical model of economic growth which encompasses total, average and relative capital and labor contribution to production growth as well the role of saving in the capital accumulation process, as we propose in sections four and five.

In his critique of Harrod (1939), Solow (1956, p. 65) argued that “all theory depends on assumptions which are not quite true” and “a ‘crucial’ assumption is one on which the conclusions do depend sensitively, and it is important that crucial assumptions be reasonably realistic”. This allows

us to analyze and discuss by briefly rethinking Solow (1956) and Swan (1956)'s appraisal of Harrod (1939), which is the aim of section six. Finally, we state the conclusions and implications.

Natural and Warranted Growth Rate

Harrod (1939) suggested that $Y(t)$ is the *existing level* of income or output, so that $\dot{Y} = dY/dt$ expresses its increment and $\lambda = (dY/dt)/Y(t)$ is the *actual rate of growth* of $Y(t)$, therefore:

$$(1) \lambda = \dot{Y}/Y(t)$$

He also defined an average *warranted rate of growth* λ_w which produces *the right amount* \dot{Y}_w at every point on the path of output.

$$(2) \lambda_w = \dot{Y}_w / Y(t)_w$$

and λ would diverge from λ_w for random or seasonal causes. In total saving $S(t) = sY(t)$, (s) "may be expected to vary, with the size of income, the phase of the trade cycle, institutional changes, etc." (Harrod, 1939, p. 16). At constant (s), it follows that $\dot{S} = dS/dt$ and $(dS/dt)/Y(t) = s(dY/dt)/Y(t)$ so that the growth of saving is equal to a fraction of the *actual growth of income*:

$$(3) \dot{S} = s\dot{Y}$$

And for the *warranted growth of output*:

$$(4) \dot{S}_w = s\dot{Y}_w$$

By equalizing equations (4) and (3), $\dot{S} = \dot{S}_w$; $\dot{Y} = \dot{Y}_w$.

Let $K(t)$ stands for the value of capital goods (both circulating and fix capital) required for $Y(t)$. Let $c = K(t)/Y(t)$ be the actual average capital cost of production and when (c) remains constant, we can derive that $dK/dt = c dY/dt \therefore c = \dot{K}/\dot{Y}$ this is "the value of the increment of capital stock in the period divided by the increment of total output" (Harrod, 1939, p. 17) and if $\dot{Y} = \dot{K}/c$, per equation (1) $\lambda c = \dot{K}/Y(t)$ and "depending on the

proposition that actual saving in a period (...) is equal to the addition to the capital stock” (Harrod, 1939, p. 18) means that $\dot{K} = sY(t)$, and thus:

$$(5) \lambda = s/c$$

From equation (3) $\dot{Y} = \dot{K}/c$ so $\dot{S} = s/c \dot{K}$ and if $\dot{K} = S(t)$ then $\dot{S} = s/c S(t)$ and letting $\dot{s} = \dot{S}/S(t)$ be the rate of growth of saving:

$$(6) \dot{s} = s/c$$

Thereby:

$$(7) \dot{s} = \lambda$$

\dot{K} measures the amount of additional capital goods to the expected stock $K(t)$, then $k = \dot{K}/K(t)$, where (k) is the rate of growth of capital, and thus $k = s Y(t)/K(t)$ from which:

$$(8) k = s/c$$

Linking this result to equation (5), $k = \lambda$; this revealed after replacing $k = \dot{K}/K(t)$ in $\lambda c = \dot{K}/Y(t)$. So far Harrod’s assumptions explain by equations (5), (6) and (8) that in a dynamic and perhaps unstable economic system $\lambda = \dot{s} = k$ is possible, where labor is assumed exogenous.

In the *warranted* rate of growth, it is supposed that $K(t)_w$ and $c_w = K(t)_w / Y(t)_w$ and c_w remaining constant yields $dK_w/dt = c_w dY_w/dt \therefore c_w = \dot{K}_w / \dot{Y}_w$ and $\dot{Y}_w = \dot{K}_w / c_w$ per equation (2):

$$(9) \dot{K}_w = c_w \lambda_w Y(t)_w$$

And if $\dot{K}_w = sY(t)_w$ it follows that:

$$(10) \lambda_w = s/c_w$$

Defining $\dot{Y}_w = \dot{K}_w / c_w$ as per equation (4) $\dot{S}_w = s/c_w \dot{K}_w$ and if $\dot{K}_w = S(t)_w$ then $\dot{S}_w = s/c_w S(t)_w$ and let $\dot{s}_w = \dot{S}_w / S(t)_w$ be the *warranted* rate of growth of saving and to have:

$$(11) \dot{s}_w = s/c_w$$

and from equation (5) it follows that:

$$(12) \dot{s}_w = \lambda_w$$

So *warranted* relative production growth could vary according to the *warranted* rate of saving growth. Furthermore $k_w = \dot{K}_w / K(t)_w$ so that $k_w = s Y(t)_w / K(t)_w$ is “conceived to vary with the current level of income, as distinct from its rate of growth” (Harrod, 1939, p. 27):

$$(13) k_w = s/c_w$$

Per equations (13) and (10) $k_w = \lambda_w$ and in a dynamic and perhaps unstable *warranted* economic system $\lambda_w = \dot{s}_w = k_w$. Equalizing equations (10) and (5) results in $\lambda_w = \lambda$; $c = c_w$, which is Harrod’s *fundamental equation*, assuming “that all new capital goods are required for the sake of the increment of output of consumer’s goods accruing” (Harrod, 1939, p. 17). This assumption requires $Y(t) = Y(t)_w$; at $c = c_w$ per equation (9) $c\lambda_w = \dot{K}_w / Y(t)_w$ and if $K_w = s Y(t)_w$ then $\lambda_w = s/c \therefore \lambda_w = \lambda$.

The Coefficient of Acceleration

Harrod’s *warranted* rate of growth is a subsystem at a given state of technology of the *natural* rate of growth system. Harrod’s assumption can be systematized by defining the *multiplier* coefficient; one component is:

$$(14) k = K(t)_w / K(t)$$

By replacing $K(t)_w = k K(t)$ in $c_w = K(t)_w / Y(t)_w$ it results in $c_w = k K(t) / Y(t)_w$ and:

$$(15) k = c_w Y(t)_w / K(t)$$

And $dK_w/dt = k dK/dt \therefore \dot{K}_w = k \dot{K}$ and since $\dot{K}_w = sY(t)_w$ and $\dot{K} = sY(t)$ then:

$$(16) k = Y(t)_w / Y(t)$$

a counterpart of equation (14) showing how k influences the rate of economic growth. In the case of saving $S(t)_w = sY(t)_w$ and $Y(t)_w = k Y(t)$ so $S(t)_w = skY(t)$ and:

$$(17) k = S(t)_w / S(t)$$

This is equivalent to equations (14) and (16).

Natural/warranted rates of growth converge at $\lambda Y(t) = \lambda_w Y(t)_w$ and per equation (16):

$$(18) \lambda = k \lambda_w$$

By matching equations (5) and (10) we get $\lambda/\lambda_w = c_w/c$ and let $k = c_w/c$ stand for the counterpart of equation (14). Moreover, $K_w = sY(t)_w$ it is equivalent to $kK(t) = c_w Y(t)_w$ so $s = c_w$, thus *warranted* saving corresponds to *warranted* average capital contribution to *warranted* rate of production growth. In a dynamic economic system in which $K(t)$ does not vary at a given instant, namely $\underline{K}(t)$, from equation (14) $k = K(t)_w / \underline{K}(t)$ and for sure ($0 \leq k < 1$) and *warranted* growth rate is below the *natural* growth rate and it is a warranted 'proper' if full labor force employment is assumed. For this $k \underline{K}(t) = s Y(t)_w$ and $Y(t)_w$ tending to $Y(t)$ (e.g. for random or seasonal causes, the phase of the trade cycle, institutional changes) and $K(t)_w$ is approaching $\underline{K}(t)$ and thus $c_w = k \underline{K}(t) / Y(t)_w$; replacing in this expression $k \underline{K}(t) = s Y(t)_w$ results in $c_w = s$ and $\underline{S}_w = s \lambda_w Y(t)_w$.

Additionally, let us state that labor force $L(t)$ is compounded by warranted employed $\underline{L}(t)$ and unemployed $\bar{L}(t)$ labor force, to set that:

$$(19) L(t) = \underline{L}(t) + \bar{L}(t)$$

If $\varepsilon = \underline{L}(t)/L(t)$, $\bar{L}(t)/L(t)$ the rate of *warranted* labor force is $\varepsilon = \underline{L}(t)/L(t)$; $\{\forall: (0 < \varepsilon \leq 1)\}$ at a given instant. Then $\underline{L}(t) = \varepsilon L(t)$ and $d\underline{L}/dt = \varepsilon dL/dt$ so that $\underline{L} = \varepsilon L$ and $\dot{\underline{L}} = \varepsilon n L(t)$, where $(\dot{\underline{L}}; n)$ represent, in that order, the rates of growth of $\underline{L}(t)$ and $L(t)$. At full employment ($\varepsilon = 1$; $\dot{\underline{L}} = n$) hence ($\varepsilon < 1$; $\dot{\underline{L}} < n$) exposes imbalanced market labor; $\underline{L}(t) < \varepsilon L(t)$. So, ε is the other component of the *coefficient of acceleration*.

Resource Composition in Economic Growth

At $K(t)_w = k K(t)$ and $\underline{L}(t) = \varepsilon L(t)$ the *warranted* resource composition at a given instant is:

$$(20) \hat{r}(t) = k/\varepsilon K(t)/L(t)$$

This denotes the line of *moving equilibrium* required for the *inherent tendency of the system to instability*, it is delineated by k/ε ; the *effective coefficient of acceleration* of resource composition. By replacing $K(t) = cY(t)$ in the previous equation we get:

$$(21) \hat{r}(t) = k/\varepsilon c Y(t)/L(t)$$

And k/ε appears to accelerate the average resources productivity; and $Y(t)/L(t) = 1/(1 - c)$ thereby the prior equation becomes into $\hat{r}(t) = k/\varepsilon r(t)$.

To better understand the above equation, it is necessary to specify the *natural* resource composition:

$$(22) r(t) = K(t)/L(t)$$

This equation is contained in equation (20) which simplified is $\hat{r}(t) = k/\varepsilon r(t)$ and at $k = \varepsilon$; $\hat{r}(t) = r(t)$; *warranted* and *natural* resource composition would converge. Taking the derivative of equation (21) in terms of (t) so that $d\hat{r}/dt = k/\varepsilon c[(dY/dt)/L(t) - Y(t)/L(t)(dL/dt)/L(t)]$. Let $n = (dL/dt)/L(t) = \dot{L}/L(t)$ be the rate of growth of labor force and $\dot{Y} = \lambda Y(t)$ the rate of growth of production to have $\dot{r}_w = k/\varepsilon c[\dot{Y}/L(t) - Y(t)/L(t) \dot{L}/L(t)] \therefore \dot{r}_w = k/\varepsilon c[\lambda Y(t)/L(t) - Y(t)/L(t) \dot{L}/L(t)] \therefore$

$$(23) \dot{r}_w = k/\varepsilon [c/(1 - c)](\lambda - n)$$

and $c/(1 - c)$ describes the slope of $r(t)$ and at determined instant $r(t) = c/(1 - c)$. The *fundamental equation* of warranted resource composition and production growth rate emerges:

$$(24) \dot{r}_w = (\lambda - n) k/\varepsilon r(t)$$

It depicts the trend of $\hat{r}(t)$ at changes in (n, λ) ; at $(n = \lambda; \dot{r}_w = 0)$ it results in Keynes's (1964, p. 55) *aggregate demand function*, which "relates various hypothetical quantities of employment to the proceeds which their outputs are expected to yield". At $[n > \lambda; \hat{r}(t) < r(t)]$ increased unemployment of the labor force and potential economic growth is exhibited. If $n = 0$, $\hat{r}(t)$ remains the same and $\lambda > 0$ as long as both labor force and capital are available until they run out. It is denoted in this event that $n > 0$ represents increments

in the number of workers required to restore retired workers. These results could be different at changes in k/ε induced by technological developments, among other circumstances; notable in the case that it makes $k/\varepsilon > 1$ and the economy becomes capital intensive. Under these conditions, (c) could diminish and \hat{r}_w could decline, but perhaps cushioned by augments in resource productivity λ .

Differentiating equation (20) with respect to (t) results in $d\hat{r}/dt = k/\varepsilon [(dK/dt)/L(t) - K(t)/L(t) (dL/dt)/L(t)]$ so $\hat{r}_w = k/\varepsilon [\dot{K}/L(t) - r(t) \dot{L}/L(t)]$ and given $\dot{K} = sY(t)$ and $\dot{L} = nL(t)$:

$$(25) \hat{r}_w = k/\varepsilon [sY(t)/L(t) - n r(t)]$$

Warranted resource composition follows the trajectory of *natural* resource composition at an *acceleration* at which capital requires a certain amount of labor and saving to yield a determined level of labor productivity. If increments in labor force cannot be employed at $r(t)$ given $(k/\varepsilon, s)$, it could cause $\hat{r}(t)$ to decline below $r(t)$, revealing labor force unemployment and potential economic growth. From equation (25) *two* new outcomes are possible: the first showing that *ceteris paribus* labor contribution to production defines the slope of $r(t)$ at each level of production; changes in (c) *ceteris paribus* will cause oscillations of $\hat{r}(t)$ at $r(t)$:

$$(26) \hat{r}_w = k/\varepsilon [s/(1 - c) - n r(t)]$$

This shows that if $\hat{r}_w = 0$ then $r(t) = s/n [1/(1 - c)]$. Given $\dot{K} = sY(t) \therefore kK(t) = sY(t) \therefore s = kc$ then:

$$(27) \hat{r}_w = k/\varepsilon [k c/(1 - c) - n r(t)]$$

At $\hat{r}_w = 0$; $r(t) = k/n [c/(1 - c)]$ and at $(n = k)$; $r(t) = c/(1 - c)$ which requires $s = nc = kc$ and thus $\hat{r}_w = k/\varepsilon n [r(t) - r(t)] = 0$; it explains that *ceteris paribus* if $n > 0$ inevitably [$n = (k > 0)$] to preserve $\hat{r}(t) = k/\varepsilon r(t)$.

Equations (26) and (27) are analogous to Solow's *fundamental equation* in which $k = sY(t)/K(t) = sF[K(t), L(t)]/K(t) = sF[1, 1/r(t)]$ so that $\hat{r}_w = k/\varepsilon [sF[K(t), L(t)]/K(t) r(t) - n r(t)] \therefore$

$$(28) \hat{r}_w = (k - n) k/\varepsilon r(t).$$

Suppose that $(k = 0)$ at *actual* k/ε and as per equation (28) $\dot{r}_w = -n k/\varepsilon r(t)$ and \dot{r}_w will fluctuate at $r(t)$; due to $\hat{r}(t) = k/\varepsilon r(t)$ then $\dot{r}_w = -n \hat{r}(t)$, explaining that *actual* $\hat{r}(t)$ might rise (decrease) as (n) varies and it could occur even if $s > 0$. On the other hand, if *ceteris paribus* $(n = 0)$ equation (28) becomes $\dot{r}_w = k/\varepsilon k [c/(1 - c)] = k/\varepsilon k r(t) \therefore \dot{r}_w = k \hat{r}(t)$ and variations in (k) induce fluctuations of \dot{r}_w around $r(t)$.¹ The second result requires a definition of the production growth function, which is described in the following section.

Let us introduce the above definition $\dot{r}_w(t) = \varepsilon n L(t)$ after differentiating equation (20), to have:

$$(29) \dot{r}_w = [c_w/(1 - c_w)](\lambda_w - \dot{n})$$

$c_w/(1 - c_w)$ is the slope of $\hat{r}(t)$ and $\hat{r}(t) = c_w/(1 - c_w)$ at a given instant (t) , hence:

$$(30) \dot{r}_w = (\lambda_w - \dot{n}) k/\varepsilon r(t)$$

This is Keynes's (1964, p. 55) *effective demand function*, which is:

Simply the aggregate income (or proceeds) which the entrepreneurs expect to receive, inclusive of the incomes which they will hand on to the other factors of production, from the amount of current employment which they decide to give (...) the point on the aggregate demand function which becomes effective because, taken in conjunction with the conditions of supply, it corresponds

1 Recall Solow's (1956, p. 70) assumption: "if the capital-labor ratio (...) should ever be established, it will be maintained, and capital and labor will grow thenceforward in proportion. By constant returns to scale, real output will also grow at the same relative rate n , and output per head of labor force will be constant". It does not necessarily reflect a *natural* rate of growth, because for (r) to be stable it is required that $n\dot{r} = kr \therefore n\dot{r} = sF(r, 1)$; $kr = sF(r, 1)$. At $n\dot{r} > [kr = sF(r, 1)]$ capital investment must increase proportionally to (s) to keep (r) value *ever established*; to say: $r = (s/k) F(r, 1)$ and of course $r = s F(r, 1)/s F(r, 1) = 1$ proves that $\dot{r} = 0$ and productive curve $y = F(r, 1)$ also keeps its value but $Y = F(K, L)$ and if $r = K/L$ then $Y = F(r, 1)L$ will be higher than before showing a greater level of production by the amount $dY/dt = F(r, 1)dL/dt \therefore \dot{Y} = F(r, 1)\dot{L}$. Hence, $n\dot{r} > [kr = sF(r, 1)]$ explains a new (lower) position of the *natural* rate of growth because of $n_2 > n_1$ which indicates a greater potential for economic growth, and the prior location becomes the *warranted* rate of growth denoting unemployed labor force perhaps due to shortage of capital which inhibits $\dot{r} > 0$. Solow (1957) makes an effort to better explain this issue. Additionally: "Even without technological progress, capital accumulation increases labor productivity, at least to a certain point, both because more capital is used per workman in each industry and because there is a shift of labor to industries that use more capital and can afford to pay a higher wage" (Domar E., 1946, pp. 138-139).

to the level of employment which maximises the entrepreneur's expectation of profit.

Matching equations (30) and (24), the possibility is shown for these two rates to coincide: $(\lambda - \lambda_w) = (n - \check{n})$ and if in a specific point of time $(n = \check{n})$ then $\lambda = \lambda_w$. The rate (s) plays a role in this point of equilibrium: inserting $\dot{K} = sY(t)$ and $\check{n}\check{L} = \varepsilon nL$ into $\dot{r}_w = k/\varepsilon [K/L(t) - r(t) \check{L}/L(t)]$ and after simplifying the result, $\dot{r}_w = [s/(1 - c_w) - \check{n} k/\varepsilon r(t)]$ and $\hat{r}(t) = k/\varepsilon r(t)$ in order for:

$$(31) \quad \dot{r}_w = [s/(1 - c_w) - \check{n} \hat{r}(t)]$$

Equalizing this equation with equation (26) and simplifying the result, $k/\varepsilon s/(1 - c) - s/(1 - c_w) = [n - \check{n}] \hat{r}(t)$ and at $(n = \check{n})$; $k/\varepsilon = (1 - c)/(1 - c_w)$; substituting this expression in equation (26) it is found that $s/(1 - c) = n r(t)$ and $\dot{r}_w = 0$. In addition, $s = nc$, which when replaced in the preceding definition gives $r(t) = c/(1 - c)$. Similarly, $s = \check{n}c_w$ so that equation (31) gives $\dot{r}_w = [\check{n}c_w/(1 - c_w) - \check{n} \hat{r}(t)] \therefore \dot{r}_w = [\hat{r}(t) - \hat{r}(t)]\check{n} \therefore \dot{r}_w = 0$. If $(\check{n} = k)$; $s = kc_w$ and $\dot{r}_w = [kc_w/(1 - c_w) - \check{n} \hat{r}(t)]$, then:

$$(32) \quad \dot{r}_w = [k - \check{n}] \hat{r}(t)$$

Once equations (32) and (28) are harmonized, $(k - \check{n}) \hat{r}(t) = (k - n) k/\varepsilon r(t)$, hence:

$$(33) \quad \hat{r}(t) = [(k - n)/(k - \check{n})] k/\varepsilon r(t)$$

This equation elucidates the fundamental reasons why $\hat{r}(t)$ might fluctuate around $r(t)$; at $(\check{n} = n)$; $k/\varepsilon = 1$ it is unveiled that $\hat{r}(t) = r(t)$; precisely at the point of *warranted/natural* rates of growth stability or at the equilibrium of *effective/aggregated* demand functions, when $(n = \check{n})$; $(c = c_w)$; $\dot{r}_w = 0$; $\lambda = \lambda_w$.

Warranted and Natural Production Growth Function

This theoretical analysis suggests that Harrod's dynamic economic growth basically combines capital goods namely $K(t)_w$ and labor $L(t)$ to provide a *warranted* level of production. The *warranted* production function might be:

$$(34) \quad Y(t)_w = F [kK(t), \varepsilon L(t)]$$

As per equation (20), the prior equation is transformed into:

$$(35) Y(t)_w = F [\hat{r}(t), 1] \varepsilon L(t)$$

It defines the average warranted labor productivity at a given $\hat{r}(t)$; $Y(t)_w/\varepsilon L(t) = F[\hat{r}(t), 1]$ is the warranted average labor's productivity curve, and at $\underline{L}(t) = \varepsilon L(t)$ then $Y(t)_w/\underline{L}(t) = F [\hat{r}(t), 1]$:²

$$(36) y_{\underline{L}} = F [\hat{r}(t), 1]$$

At $\underline{L}(t) = \varepsilon L(t)$ function (35) turns into an effective warranted production function $Y(t)_w = F [\hat{r}(t), 1] \underline{L}(t)$ whose derivative provides that $dY_w/dt = f [\hat{r}(t), 1] d\underline{L}/dt \therefore \dot{Y}_w = f [\hat{r}(t), 1] \dot{\underline{L}}$ which in terms of *warranted* average rate of labor's productivity is:

$$(37) \lambda_{\underline{L}} = (1 - c_w) \check{n} y_{\underline{L}}$$

ceteris paribus ($\check{n} > 0$; $\lambda_{\underline{L}} = \check{n}$) capital's and labor's productivity remains the same.

The derivative of function (32) in terms of (t) gives:

$$(38) \dot{Y}_w = f (k\dot{K}, \varepsilon\dot{L})$$

And if $\dot{K} = sY(t)$ at a determined point of production processes, the previous function is $\dot{Y}_w = f [ksY(t), \varepsilon\dot{L}]$ and per equation (16) $Y(t)_w = k Y(t)$ so that $\dot{Y}_w = f [sY(t)_w, \varepsilon nL(t)]$ and since $\check{n}\dot{\underline{L}}(t) = \varepsilon nL(t)$ then:

$$(39) \dot{Y}_w = f [sY(t)_w, \check{n}\dot{\underline{L}}(t)]$$

It follows that $\lambda_w Y(t)_w = f [sY(t)_w, \check{n}\dot{\underline{L}}(t)]$, so:

$$(40) \lambda_w = f [s, (1 - c_w)\check{n}]$$

At a warranted rate of growth, it is a condition $s = \check{n}c_w$ which substituted in the above function, can be useful "when production takes place under the

2 In contrast Solow's productivity curve (1956, pp. 69-71) depicts *natural* labor productivity according to Harrods' definition (1939, p. 30).



usual neoclassical conditions of variable proportions and constant returns to scale” (Solow, 1956, p. 73):

$$(41) \lambda_w = f(\tilde{n}) = f(s/c_w)$$

Let this be the *warranted* neoclassical condition, which is also Keynes’s *effective demand*; if $\lambda_w = 0$ in function (40) then $|\tilde{n}/s| = [1/(1 - c_w)]$ and if $s = \tilde{n}c_w$ then $\hat{r}(t) = [c_w/(1 - c_w)]$. After inserting $\tilde{n} = s/c_w$ in function (40) another result is: $\lambda_w = f[1, 1/\hat{r}(t)]s$ where:

$$(42) y_K = F[1, 1/\hat{r}(t)]$$

If $(\tilde{n} = k) = s/c_w$ then $s = kc_w$, so that:

$$(43) \lambda_K = kc_w y_K$$

If $y_L = y_K \therefore F[\hat{r}(t), 1] = F[1, 1/\hat{r}(t)]$ the result will be $\hat{r}(t) = 1$ and by equalizing equations (43) and (37) $\tilde{n}/k = c_w/(1 - c_w)$ where $\mu_w = \tilde{n}/k$ is the *elasticity* or *velocity* of $\hat{r}(t)$, so that:

$$(44) \mu_w = c_w/(1 - c_w)$$

Now it is feasible to measure c_w at each instant:

$$(45) c_w = \mu_w/(1 + \mu_w)$$

When production processes are labor intensive it occurs that $(0 < \mu_w < 1)$ and capital contribution to production growth could be lesser than that of labor. In a capital-intensive production process $(\mu_w > 1)$ and capital contribution will be greater than that of labor.

When $\dot{K} = kK(t)$ function (38) reveals *warranted* production growth at both $K(t)_w$ and $L(t)$ levels: $\dot{Y}_w = f(k kK(t), \epsilon L)$. Also $\dot{n}L(t) = \epsilon nL(t)$ and $K(t)_w = k K(t)$, hence:

$$(46) \dot{Y}_w = f[kK(t)_w, \dot{n}L(t)]$$

Equalizing this function with function (39) results in $kK(t)_w = sY(t)_w$ which proves that $s = kc_w$, as suggested before. Function (46) can also be expressed as $\lambda_w Y(t)_w = f [kK(t)_w, \check{n}L(t)]$ and $\lambda_w = f [kK(t)_w / Y(t)_w, \check{n}L(t) / Y(t)_w]$ so that:

$$(47) \lambda_w = f [kc_w, (1 - c_w)\check{n}]$$

This also arises from function (40) after substituting $s = kc_w$. Multiplying both sides of that function by $Y(t)_w$ results in $\lambda_w Y(t)_w = f [kc_w, (1 - c_w)\check{n}]Y(t)_w$ and $\dot{Y}_w = [kc_w Y(t)_w, (1 - c_w)\check{n} Y(t)_w]$ reveals total contribution of capital and labor to the value of \dot{Y}_w . For the additional capital it is:

$$(48) \dot{Y}_{K(t)_w} = kc_w Y(t)_w$$

where $kc_w = \dot{Y}_{K(t)_w} / Y(t)_w$ and let $\lambda_{K(t)_w} = \dot{Y}_{K(t)_w} / Y(t)_w$ be the average contribution of capital, so that:

$$(49) \lambda_{K(t)_w} = kc_w$$

And the relative contribution is:

$$(50) \lambda_{K(t)_w} / k = c_w$$

For the added labor total contribution is:

$$(51) \dot{Y}_{L(t)} = (1 - c_w)\check{n}Y(t)_w$$

And $\lambda_{L(t)} = \dot{Y}_{L(t)} / Y(t)_w$ is its average contribution, from which:

$$(52) \lambda_{L(t)} = (1 - c_w)\check{n}$$

and its relative contribution is:

$$(53) \lambda_{L(t)} / \check{n} = (1 - c_w)$$

Per equations (49) and (52) which are equivalents of equations (43) and (37), respectively, function (47) is redefined as:

$$(54) \lambda_w = f [\lambda_{K(t)_w}, \lambda_{L(t)}]$$

The *warranted* rate of growth is influenced by the fundamental conditions of the *natural* rate of growth. In function (38) $\lambda_w Y(t)_w = f [kK(t), \varepsilon nL(t)]$ and if $\check{L}(t) = \varepsilon L(t)$ and $K(t)_w = kK(t)$ then:

$$(55) \lambda_w Y(t)_w = f [k K(t)_w, n \check{L}(t)]$$

This function makes visible the available capital and labor force effectively employed in a specific period of production. Per this new function $\lambda_w = f [k K(t)_w / Y(t)_w, n \check{L}(t) / Y(t)_w]$ and thus:

$$(56) \lambda_w = f [kc_w, (1 - c_w)n]$$

This function is useful in depicting dynamic instability in the economic growth system due to the fact that (k, n) are related to the natural increments of capital and labor force, and for this reason might reveal the unemployment of those available resources at k/ε . Suppose $\lambda_w = 0$, so:

$$(57) | -n/k | \begin{matrix} \geq \\ \leq \end{matrix} c_w / (1 - c_w)$$

In this case $\mu = n/k$ is the *elasticity* or *velocity* of natural resource composition, which should be greater or lesser than or equal to the *velocity* of warranted resource composition μ_w ; $(\mu = n/k) \begin{matrix} \geq \\ \leq \end{matrix} (\mu_w = \check{n}/k)$. This will depend on the *momentums* of the economic growth system, where *movements* are depicted by the product *mass* by *velocity* and (μ, μ_w) reflects the oscillating movements in the *gravitational field* of economic growth (\dot{Y}, \dot{Y}_w) tracked by (n, \check{n}, k) and accelerated by k/ε .

Natural momentum: $\dot{Y} = [\dot{K}, \dot{L}] \therefore \lambda Y(t) = [kK(t), nL(t)]$ where the *masses* are $Y(t), K(t), L(t)$ so that the respective velocities are λ, k, n from which $Y(t) = F [K(t), L(t)]$ in *momentum zero* and $\lambda Y(t)$ is the variation of $Y(t)$ at λ velocity as a result of the velocity (k) by mass $K(t)$ for capital and velocity (n) by mass $L(t)$ for labor force: $\lambda = f(\mu)$. In this case, some *centrifugal forces* include ignition of a field in motion toward instability, increase in population, accumulation of capital and savings, science, technological improvement and the work/leisure preference schedule (Harrod R. F., 1939, p. 30). *Warranted momentum:* $\dot{Y}_w = f (k\dot{K}, \varepsilon\dot{L}) = \lambda_w Y(t)_w = f [kK(t), \varepsilon nL(t)]$ is the product of such masses by such velocities (\check{n}, k) whose outcome is *warranted* economic growth: $\lambda_w = f(\mu_w)$. It is a *moving equilibrium* caused by the circumstances operating as *centripetal* and other inertial

forces, conjugated by (k/ε) ; including government policies (e.g. monetary, fiscal, commerce), the trade cycle and level of activity, producer and labor organizations and rules (Harrod R. F., 1939, p. 22).

These two *momentums* can be exposed by conjunction:
 $\lambda Y(t) = \lambda_w Y(t)_w \therefore [kK(t), nL(t)] = [kkK(t), \varepsilon nL(t)]$ so that
 $(1 - k)kK(t) = - (1 - \varepsilon)nL(t)$ and thus $| - n/k | = [(1 - k)/(1 - \varepsilon)] K(t)/L(t)$ hence:

$$(58) \mu = (1 - k)/(1 - \varepsilon) r(t)$$

In that system $\mu = \mu_w = r(t)$ at $(\varepsilon = k)$ *ceteris paribus* but if $(\varepsilon \neq k)$ the previous equation turns into:

$$(59) \mu_w = (1 - k)/(1 - \varepsilon) r(t)$$

Which is $\mu_w = \hat{r}(t)$: “The acceleration principle is presented as a leading dynamic determinant” but “there is no inherent tendency for these two [*momentums*] to coincide” (Harrod R. F., 1939, pp. 26, 30). The role of $\mu = n/k$ can be clarified by equation (56) by $n = \mu k$ or $k = n/\mu$ and:

$$(60) \lambda_w = f [c_w, (1 - c_w)\mu]k; \lambda_w = f [c_w/\mu, (1 - c_w)]n$$

The natural production function can also be defined simply by Solow’s production function:

$$(61) Y(t) = F[K(t), L(t)]$$

whose derivative is $\dot{Y} = (\dot{K}, \dot{L})$ so $\lambda = f [k K(t)/Y(t), n L(t)/Y(t)]$ and the *natural* rate of growth is:

$$(62) \lambda = f [ck, (1 - c)n]$$

And if $(n = k)$; $\dot{r} = 0$ we get the equivalent of function (41) which is also Keynes’s *aggregated demand*:

$$(63) \lambda = f(n) = f(\dot{r} - k); \lambda = f(k) = f(\dot{r} - n)$$

At $\dot{r} = 0$ and $(k; n) > 0$ it implies a diminishing contribution. At $\lambda = 0$ in equation (62):

$$(64) \quad | - n/k | = c/(1 - c)$$

This exposes the reasons why equation (57) is expressed as inequality. Also:

$$(65) \quad \mu = c / (1 - c)$$

$$(66) \quad c = \mu / (1 + \mu)$$

Let $c = F(\mu)$ to obtain 1): $\lim_{\mu \rightarrow 0} F(\mu) = \lim_{\mu \rightarrow 0} \left(\frac{\mu}{1 + \mu} \right) \approx 0$ and 2) $\lim_{\mu \rightarrow \infty} F(\mu) = \lim_{\mu \rightarrow \infty} \left(\frac{\mu}{1 + \mu} \right) \approx 1$ to conclude that $\{c:(0,1)\}$ thereby (c) descends horizontal asymptote as $\mu > 0$.

Saving and Capital Accumulation

Inserting $\mu_w = \check{n}/k \therefore \check{n} = k\mu_w$ into function (47) results in $\lambda_w = f [c_w, (1 - c_w)\mu_w] k$ where $\lambda_w = f [c_w, (1 - c_w)\mu_w] s/c_w$ and thus $\lambda_w = 2s$ at *constant returns to scale*. But we can compute the rate of saving by equation (47) after differentiating equation (20) to obtain $\check{r} = k - \check{n}$ so that $\lambda_w = f [(\check{r} + \check{n})c_w, (1 - c_w)\check{n}]$ from which $\lambda_w = f [(\check{r} + \check{n})c_w, (1 - c_w)\check{n}]$ and by $k = \check{n}/\mu_w$ will result $\check{r} = \check{n}/\mu_w - \check{n} \therefore \check{r} = [(1 - \mu_w)/\mu_w]\check{n}$ in order to $\lambda_w = f [(1 - \mu_w)c_w/\mu_w + 1]\check{n}$ where $\lambda_w = f [(c_w, (1 - c_w)\mu_w)/\mu_w]\check{n}$ and because $\check{n} = k\mu_w$

$$(67) \quad \lambda_w = f [c_w, (1 - c_w)\mu_w] k$$

And defining $K(t)_w = s Y(t)_w$ so that:

$$(68) \quad k = s\check{\lambda}/c_w$$

be the *effective* or *recorded* capital growth rate where $\check{\lambda}$ symbolizes the *recorded* production growth rate. This function turns into:

$$(69) \quad \lambda_w = f [c_w, (1 - c_w)\mu_w] s\check{\lambda}/c_w$$

Here (s) can be disclosed as follows:

$$(70) \quad s = [c_w / [c_w, (1 - c_w)\mu_w] \lambda_w / \check{\lambda}$$

(s) is the *warranted* rate of saving to get λ_w at each instant, which must be compared with the *effective* λ ; this outcome is also obtained per equation (68). Multiplying both sides of this equation by the *recorded* $Y(t)$ the result is the level of saving $S(t)$ at each instant. From this results must be possible to derive the capital depreciation and the replacement and new capital into capital accumulation process (Villalobos C. D., 2020). If $\lambda < \lambda_w$ the economy makes inefficient use of saving resulting in an excess of capital accumulation and low depreciation.

Steady Natural Rate of Growth

“A steady rate of increase implies that a constant fraction of income is saved. If the fraction of income saved is increasing or decreasing, that implies an accelerating or decelerating increase of income” (Harrod R. F., 1960). By inserting Harrod’s (1939) definition $\dot{K} = sY(t)$, from which $k = sY(t)/K(t)$, into equation (62) we get Swan’s (1956) *basic formula* $y = (\alpha sY/K, \beta n)$, which illustrates “the connection between capital accumulation and the growth of the productive labor force” (Swan T. W., 1956, p. 334) measured by resources’ share (income) on production growth at constant returns to scale following Cobb and Douglas’s (1928) production function, instead of resources’ *contribution to* production growth; from function (62) $\lambda = f [scY(t)/K(t), (1 - c)n]$ after substituting the above definition we obtain the *natural* economic growth rate:

$$(71) \lambda = f [s, (1 - c)n]$$

Inserting $s = nc$ into the above function we get $\lambda = f(n)$ and if $n = k$ then $\lambda = f(k)$. Also, this function provides a key result after inserting $n = s/c$:

$$(72) \lambda = s f [1, 1/r(t)]$$

This explains that increments in production per unit of capital to labor are determined by the appropriate level of saving. This previous function is a key component in both Solow’s (1956) *fundamental equation* and Swan’s (1956, p. 335) “basic formula for the rate of growth of output”. However, Swan’s basic formula provides an essential relationship between resources’ share on production growth (α), “when production takes place under the usual neoclassical conditions of variable proportions and constant returns to scale” (Solow, 1956, p. 73) and resources’ contribution to production

growth (c); $y = (\alpha sY/K, \beta n)$ and $c = K/Y$ so $y = (\alpha s/c, \beta n)$ and by adding $n = s/c$ Swan's basic formula becomes³:

$$(73) y = (\alpha, \beta)s/c$$

Swan's basic formula can be expressed as $y = (\alpha k, \beta n)$ which is the counterpart of function (62)⁴; provided that $\alpha k = kc \therefore \alpha = c$, function (43) is $\lambda = f[\alpha k, (1 - \alpha)n]$ and at $k = n = s/c$ results in $\lambda = f[\alpha, (1 - \alpha)]s/c$:

$$(74) \lambda = f(s/c)$$

It also explains that rate (s) is required for steady λ ; $s = f(c\lambda)$ which can be obtained from equation 62 or 63. From function (56) the corresponding warranted rate of economic growth is $\lambda_w = f[c_w sY(t)/K(t), (1 - c_w)n]$ so that:

$$(75) \lambda_w = f[c_w s/c, (1 - c_w)n]$$

Or also:

$$(76) \lambda_w = f[\alpha_w, (1 - \alpha_w)]s/c; \lambda_w = f(s/c)$$

And for Swan's basic formula: $y = (\alpha_w, \beta_w)s/c$.

Equation (28) can, therefore, be renewed as follows:

$$(77) \dot{r}_w = k/\varepsilon [c/(1 - c) - r(t)] s/c$$

This reveals the tendency of warranted resource composition as *ceteris paribus* the rate of saving varies, denoting an unbalanced production growth. Per equation (24), that condition of economic growth instability can be expressed as $\dot{r}_w = (\lambda - n) \hat{r}(t)$ and at $n = s/c$ will result in:

3 In Swan's (1956) *basic formula*, any point (sY/K) represents Harrod's *warranted* rate of growth and any point in (y) corresponds to Harrod's *natural* rate of growth: so, *at an equilibrium point* $y = sY/K = s/c$.

4 Per Cobb and Douglas's (1928) production function, on which Swan's (1956) *unclassical case* is based, it can be proven that $r_s = \alpha k = rck$; $Y = F[K^\alpha L^{1-\alpha}]$ and by partial derivative in terms of K so that $\partial Y/\partial K = f[\alpha K^{\alpha-1} L^{1-\alpha}]$ and let $r = \partial Y/\partial K$ be the rate of or capital return to yield $r = \alpha f(Y/K) \therefore r = \alpha/c$ and thus $\alpha = rc$; by replacing this result in function (73), we get $y = (rs, \beta n)$, as per Swan's (1956, p. 335) footnote No. 5.

$$(78) \dot{r}_w = (\lambda - s/c) k/\varepsilon r(t)$$

It is clear that the natural rate of growth is determined by *velocities*, the rate of saving and the average capital's contribution. Harrod's instability of economic growth motivates Solow's (1956) 'A model of long-run growth' providing the *fundamental equation* $\dot{r} = s F[r(t), 1] - n r(t)$, which describes the potential trajectory of $r(t)$. Solow's equation comes from the derivative of natural resource composition according to equation (23): $dr/dt = [(dK/dt)/L(t) - K(t)/L(t) (dL/dt)/L(t)] \therefore \dot{r} = [\dot{K}/L(t) - K(t)/L(t) \dot{L}/L(t)]$ and $\dot{r} = [k K(t)/L(t) - n K(t)/L(t)]$ so that $\dot{r} = k r(t) - n r(t)$ and after replacing $k = sY(t)/K(t)$ results in $\dot{r} = s/c r(t) - n r(t)$ and if $n = s/c$:

$$(79) \dot{r} = [r(t) - r(t)]s/c$$

This equation is tautological, reason for which it reveals only a steady natural rate of growth in Solow's fundamental equation. By partially reverting Solow's equation it will yield $Y(t)/K(t) = F[[K(t), L(t)]/K(t)] = F[r(t), 1] = 1/c$ and we can rewrite it as follows:

$$(80) \dot{r} = s/c - n r(t)$$

which can be achieved for labor by following the previous process: $\dot{r} = [\dot{K}/L(t) - n r(t)]$ and then substituting $\dot{K} = sY(t)$ to get the counterpart of equation (26):

$$(81) \dot{r} = [s [1/(1 - c)] - n r(t)]$$

And $s = nc$:

$$(82) \dot{r} = [n [c/(1 - c)] - n r(t)]$$

And $\mu = c/(1 - c)$ is a result of the derivative of Solow's production function $Y(t) = F[K(t), L(t)]$ to achieve function (62) and at $\lambda = 0$:

$$(83) \dot{r} = [\mu - r(t)] s/c$$

At $[\mu = 1; \dot{r} = 0; \lambda = f(\dot{r}) = 0]$ and if $k = \varepsilon; \mu = \hat{r}(t)$.

Conclusions

As the introduction claims, the purpose of this study is to provide a better understanding of Harrod's dynamic theory and to offer a *new method of approach to attack* some gaps. By working on the fundamental assumptions and suggestions supporting Harrod's basic concepts and equations, we devised a new set of relations that provides an analytical model for dynamic economic growth. An interesting contribution of this study is having found new formulas with which to measure the coefficient of acceleration, resource composition, the rate of thrift for the require capital accumulation at each instant and their influence in the instability of warranted and natural production function. We also derived *velocity* as the *elasticity* of resource composition with which precise capital and labor contribution to production growth (Villalobos C. D., 2019; 2019a; 2020).

We conclude that warranted and natural rates of growth belong to the same system but they do not necessarily converge due to the fact that full employment of resources might be difficult to achieve. When convergence between these rates of growth occurs it appears as a *moving equilibrium* depending on the velocities of warranted and natural rates of growth in capital and labor, the *state of the art or technique* acting as the *accelerator* of warranted resource composition, and the capital and saving accumulation, among other centrifugal and centripetal social forces.

In such a system, Solow's (1956, p. 73) basic conclusion could occur as a moving equilibrium where momentarily the system can adjust at the rate of growth of the labor force but not *approach a state of steady proportional expansion*. Cobb and Douglas (1928) questioned whether the increase in production is *purely fortuitous* or is primarily caused by technique, and the degree to which it responds to changes in the quantity of labor or capital. This strongly asserts Harrods' natural and warranted rates of growth divergence, because natural rates of growth could be influenced by *random* events, but the warranted rate could assure that the processes of distribution are modeled at all closely upon those of the production of value, as suggested by Cobb and Douglas (1928). It cannot occur only 'by a fluke' (Swan T. W., 1956, p. 343), not as an odd piece of luck but as a consequence of demand-supply adjustments (Solow, 1956, p. 77).

We believe that these results could contribute to improving economic scientific knowledge on economic growth and be useful in enhancing government macroeconomic policies for a better allocation of available economic resources. This, in turn, will help to maintain a dynamic yet stable warranted rate of economic growth, thereby making business decisions less risky. New research in this field must pay explicit attention to variables such as the rate of interest, capital and labor costs and product pricing to achieve advances based on in this new model.

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