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A Appendix

A.1 Cantiliver design problem [8]

The Cantilever beam is made of five elements, each having a hollow cross-section with constant thickness. The beam is rigidly supported as shown, and three is an external vertical force acting at the free end of the cantilever. The weight of the beam is to be minimized while assigning an upper limit on the vertical displacement of the free end.

The design variables are the heights (or widths) x_i of the cross-section of the each element. The lower bounds on the these design variables are very small and the upper bounds very large so they do not become active in the problem. The problem is formulated as follows:

$$Minimize\;\; f(x) = 0.0624(x_1 + x_2 + x_3 + x_4 + x_5)$$

s.t.

$$g_1(x) = \frac{61}{x_1^3} + \frac{37}{x_2^3} + \frac{19}{x_3^3} + \frac{7}{x_4^3} + \frac{1}{x_5^3} \leq 1.0$$

$$1 \leq x_i \leq 10, i \in \{1, 2, 3, 4, 5\}.$$

Solution found:

$$x^* = (5.80832436167656592, 2.88233457568314051, 4.21582930749505342,$$

$$3.44602689729287517, 2.08988145846961546).$$

$$F^* = 1.150805547878516.$$

A.2 Two-bar truss design problem [8]

The two-bar truss problem consist of two design variables: a sizing variable x_1 which is the cross-sectional area of the bars and the configuration variable x_2 representing half the distance between the lower nodes. An external force, $|F| = 200kN$, $F_y = 8F_x$, acts on node 3 and the objective is to minimize the weight of the truss while keeping the tensile or compressive stress in each bar below $100N/mm^2$. The problem is formulated in closed form as:

$$Minimize\ \ f(x) = x_1\sqrt{1+x_2^2}$$

s.t.

$$g_1(x) = 0.124\sqrt{1+x_2^2}\left(\frac{8}{x_1}+\frac{1}{x_1x_2}\right) \leq 1.0\ (bar1),$$

$$g_2(x) = 0.124\sqrt{1+x_2^2}\left(\frac{8}{x_1}-\frac{1}{x_1x_2}\right) \leq 1.0\ (bar2),$$

$$0.2 \leq x_1 \leq 4.0,\ \ 0.1 \leq x_2 \leq 1.6.$$

Solution found:

$$x^* = (1.41274204233180889, 0.37472108515071976)$$

$$F^* = 1.508670852887466.$$

A.3 Three-bar truss design problem [29]

The three-bar truss problem consist of two design variables: The volume of the truss structure is to be minimized subject to stress constraints. The problem is formulated as:

$$Minimize\ \ f(x) = \left(2\sqrt{2}x_1 + x_2\right\} L$$

s.t.

$$g_1(x) = \left(\frac{\sqrt{2}x_1 + x_2}{\sqrt{2}x_1^2 + 2x_1x_2}\right) P \leq 2,$$

$$g_2(x) = \left(\frac{1}{x_1 + \sqrt{2}x_2}\right) P \leq 2,$$

$$g_3(x) = \left(\frac{2}{\sqrt{2}x_1^2 + 2x_1x_2}\right) P \leq 2,$$

where $0 \leq x_1 \leq 1$ and $0 \leq x_2 \leq 1$. The other constants are $L = 100$cm, $P = 2kN/cm^2$.
Solution found:

$$x^* = (0.79271422810570653, 0.39694263279557871).$$

$$F^* = 263.90770577419977.$$

A.4 Welded beam design problem [11]

A welded beam design optimization problem, which is often used for the evaluation of optimization methods, is used to illustrate the implementation procedure of the proposed approach for solving optimization problems. The beam has a length of 14 in. and P=6,000 lb force is applied at the end of the beam. The welded beam is designed for minimum cost subject to constraints on shear stress, bending stress in the beam, buckling load on the bar, end deflection of the beam, and side constraints. The design variables are thickness of the weld $h(x_1)$, length of the weld $l(x_2)$, width of the beam $t(x_3)$, and thickness of the beam $b(x_4)$. The mathematical model of the welded beam optimization problem is defined as

$$Minimize \;\; f_w(x) = 1.1047x_1^2x_2 + 0.04811x_3x_4(14.0 + x_2)$$

s.t.

$$g_1(x) = 13,600 - \tau(x) \geq 0,$$
$$g_2(x) = 30,000 - \sigma(x) \geq 0,$$
$$g_3(x) = x_4 - x_1 \geq 0,$$
$$g_4(x) = 0.10471(x_1^2) - 0.04811x_3x_4(14.0 + x_2) + 5.0 \geq 0,$$
$$g_5(x) = x_1 - 0.125 \geq 0,$$
$$g_6(x) = 0.25 - \delta(x) \geq 0,$$
$$g_7(x) = P_c(x) - 6,000 \geq 0,$$
$$0.1 \leq x_1,\;\; x_2 \leq 5$$
$$0.1 \leq x_3,\;\; x_4 \leq 10.$$

The terms $\tau(x)$, $\sigma(x)$, $P_c(x)$, $\delta(x)$ are given below

$$\tau(x) = \sqrt{(\tau')^2 + (2\tau'\tau'')\frac{x_2}{2R} + (\tau'')^2}$$

$$\tau'(x) = \frac{6000}{\sqrt{2}x_1 x_2}$$

$$\tau''(x) = \frac{6000(14 + \frac{x_2}{2})\sqrt{0.25(x_2^2) + ((x_1 + x_3)/2)^2}}{2[x_1 x_2 \sqrt{2}(x_2^2/12 + 0.25(x_1 + x_3)^2)]}$$

$$\sigma(x) = \frac{504,000}{x_3^2 x_4}$$

$$\delta(x) = \frac{65,856,000}{(30 \times 10^6) x_4 x_3^3}$$

$$P_c(x) = \frac{4.013(30 \times 10^6)\sqrt{\frac{x_3^2 x_4^6}{36}}}{196}\left(1 - \frac{x_3\sqrt{\frac{30\times 10^6}{4(12\times 10^6)}}}{28}\right).$$

Solution found:

$$x^* = (0.20586359479354222, 3.46334453602819113,$$

$$9.04774674254588592, 0.20586428001618971).$$

$$F^* = 1.727036254666027.$$

A.5 Weight tension/compression spring problem [2] [5]

This problem minimizes the weight of a tension/compression spring, subject to constraints of minimum deflection, shear stress, surge frequency, and limits on outside diameter and on design variables. There are three design variables: the wire diameter x_1, the mean coil diameter x_2, and the number of active coils x_3. The mathematical formulation of this problem is:

$$Minimize \;\; f(x) = (x_3 + 2)x_2 x_1^2$$

s.t.

$$g_1(x) = 1 - \frac{x_2^3 x_3}{71785 x_4^4} \leq 0,$$

$$g_2(x) = \frac{4x_2^2 - x_1 x_2}{12566(x_2 x_1^3 - x_1^4)} + \frac{1}{5108 x_1^2} - 1 \leq 0,$$

$$g_3(x) = 1 - \frac{140.45 x_1}{x_2^2 x_3} \leq 0,$$

$$g_4(x) = \frac{x_2 + x_1}{1.5} - 1 \leq 0,$$

Solution found:

$$x^* = (0.05044713178541634, 0.32746441361099429, 13.23998350856038107)$$

$$F^* = 0.012700521857.$$

A.6 Pressure vessel design (six inequalities)

The pressure vessel design, was previously analysed by Sandgren [32] who first proposed this problem to minimize the total cost of the material, forming and welding of a cylindrical vessel. There are four design variables: x_1 (Ts, shell thickness), x_2 (Th, spherical head thickness), x_3 (R, radius of cylindrical shell) and x_4 (L, shell length). Ts (=x_1) and Th (=x_2) are integer multipliers of 0.0625 in. In accordance with the available thickness of rolled steel plates, and R (=x_3) and L (=x_4) have continuous values of $40 \leq R \leq 80$in. and $20 \leq L \leq 60$in., respectively. The mathematical formulation of the optimization problem is as follows:

$$Minimize \;\; f(x) = 0.6224x_1x_2x_3 + 1.7781x_2x_3^3 + 3.1611x_1^2x_4 + 19.84x_1^2x_3$$

s.t.

$$g_1(x) = 0.0193x_3 - x_1 \leq 0,$$

$$g_2(x) = 0.00954x_3 - x_2 \leq 0,$$

$$g_3(x) = 750.0 \times 1728.0 - \pi x_3^2 x_4 - \frac{4}{3}\pi x_3^2 \leq 0,$$

$$g_4(x) = x_4 - 240.0 \leq 0,$$

$$g_5(x) = 1.1 - x_1 \leq 0,$$

$$g_6(x) = 0.6 - x_2 \leq 0,$$

Solution found:

$$x^* = (1.125, 0.625, 58.2900704783923345, 43.6931234586562753)$$

$$F^* = 7197.73412633523851.$$

A.7 Pressure vessel design (four inequalities)

Another variation of this problem, that has two inequalities minus (g_5 and g_6 are eliminated) and the following bounds $1\times 0.0625 \leq x1, x2 \leq 99\times 0.0625, 10.0 \leq x3, x4 \leq 200.0$ has been solved by others researchers.
Solution found by our approach:

$$x^* = (0.875, 0.4375, 45.3366721064070408, 140.255022911949085).$$

$$F^* = 6090.53937693476024.$$

A.8 Speed reducer design (continuous-integer variables)

The design of the speed reducer [20], is considered with the face width x_1, module of teeth x_2, number of teeth on pinion x_3, length of the first shaft between bearings x_4, length of the second shaft between bearings x_5, diameter of the first shaft x_6, and diameter of the first shaft x_7 (all variables continuous except x_3 that is integer). The weight of the speed reducer is to be minimized subject to constraints on bending stress of the gear teeth, surface stress, transverse deflections of the shafts and stresses in the shaft. The problem is formulated as follows:

$$Minimize\ \ f(x) = 0.7854x_1x_1^2(3.3333x_3^2 + 14.9334x_3 - 43.0934)$$

$$-1.508x_1(x_6^2 + x_7^2) + 7.4777(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2)$$

s.t.

$$g_1(x) = \frac{27}{x_1x_2^2x_3} - 1 \leq 0,$$

$$g_2(x) = \frac{397.5}{x_1x_2^2x_3^2} - 1 \leq 0,$$

$$g_3(x) = \frac{1.93x_4^3}{x_2x_3x_6^4} - 1 \leq 0,$$

$$g_4(x) = \frac{1.93x_5^3}{x_2x_3x_7^4} - 1 \leq 0,$$

$$g_5(x) = \frac{1.0}{110.0x_6^3}\sqrt{\left(\frac{745.0x_4}{x_2x_3}\right)^2 + 16.9\times 10^6} - 1 \leq 0,$$

$$g_6(x) = \frac{1.0}{85.0x_7^3}\sqrt{\left(\frac{745.0x_5}{x_2x_3}\right)^2 + 157.5\times 10^6} - 1 \leq 0,$$

$$g_7(x) = \frac{x_2 x_3}{40} - 1 \leq 0,$$

$$g_8(x) = \frac{5x_2}{x_1} - 1 \leq 0,$$

$$g_9(x) = \frac{x_1}{12x_2} - 1 \leq 0,$$

$$g_{10}(x) = \frac{1.5x_6 + 1.9}{x_4} - 1 \leq 0,$$

$$g_{11}(x) = \frac{1.1x_7 + 1.9}{x_5} - 1 \leq 0,$$

wuth $2.6 \leq x_1 \leq 3.6$, $0.7 \leq x_2 \leq 0.8$, $17 \leq x_3 \leq 28$, $7.3 \leq x_4 \leq 8.3$, $7.8 \leq x_5 \leq 8.3$, $2.9 \leq x_6 \leq 3.9$, $5.0 \leq x_7 \leq 5.5$.

Solution found by our approach:

$$x^* = (3.50002615416866586, 0.70000523059661887, 17, 7.30022922985589972,$$

$$7.8000228842193966, 3.35021507672250302, 5.28669973187709912).$$

$$F^* = 2996.3951944729081.$$

A.9 Speed reducer design (integer-discrete variables)

Another variation of this problem with the variables defined as follows: x_1, x_2, x_4, and x_5 must be integral multiples of 0.1. x_6 and x_7 must be integral multiples of 0.01, and x_3 must be an integer.

Solution found by our approach:

$$x^* = (3.3, 0.7, 17.0, 7.3, 7.8, 3.36, 5.29).$$

$$F^* = 2922.43527186608.$$